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INTRODUCTION

TO

ARITHMETIC.

WILLIAM AND ROBERT CHAMBERS,

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1854.

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NOTICE.

IN this introductory treatise on Arithmetic, the utmost simplicity of language has been studied in expressing the rules; and in every instance their meaning is shewn by examples wrought out at length before the eye of the pupil, accompanied with detailed explanations of the mode of working the questions.

In the present edition, the exercises and explanatory notes have been greatly extended, and many new rules have been added, that were not given in the former edition; such as Equation, Partnership, the Square and Cube Roots, Duodecimals, &c. The work will be found to contain all the ordinary rules taught at school.

An account of the proposed Decimal System of reckoning money, with examples and exercises, has been given as an Appendix.

CONTENTS.

INTRODUCTION TO ARITHMETIC.

ARITHMETIC is the art of counting or reckoning by means of numbers.

Number expresses either a *unit*—that is, one of anything—or a collection of units of the same kind, as *two* horsemen, *five* books, a *thousand* feet.

NOTATION AND NUMERATION.

NOTATION is the method of expressing numbers by means of certain signs or figures; thus—1, 2, 3.

NUMERATION is the art of reading or expressing numbers in words; thus—1, one; 2, two.

THE FIGURES used to express numbers are the following:—

one,	two,	three,	four,	five,	six,	seven,	eight,	nine,	nothing or nought.
1	2	3	4	5	6	7	8	9	0

The first nine of these figures, when standing separately or singly, thus—3, 6, are termed *units*, and each of them represents one of the numbers from *one to nine*. The last, 0, called a nothing or nought, when standing by itself, expresses no number, or nothing; but when annexed to any of the other figures, it increases their value tenfold. Thus 1 with 0 annexed becomes 10, or ten; 2 with 0 annexed, thus—20, represents twenty; 10 with 0 annexed represents one hundred; and so on.

These figures, besides the value they have when standing singly, have also, when standing in connection with other figures, a *local* value, depending on the *place* they occupy in the number; thus 2 by itself means two, but if it becomes the second figure from the *right* by another figure having been placed after it, thus—23, the 2 counts as 2 tens, or *twenty;* and if a third figure is annexed, so as to make it the third from the right, thus—236, the 2 counts as 2 hundreds; and so on, the addition of each figure increasing tenfold the local value of those before it. It is by means of these ten figures and their combinations that all numbers are expressed.

Numbers *ten to ninety-nine*, are expressed by the joining of *two* figures; thus—10, ten; 20, twenty; 85, eighty-five; 99, ninety-nine.

Numbers *one hundred to nine hundred and ninety-nine*, are expressed by the joining of *three* figures; thus—100, one hundred; 500, five hundred; 867, eight hundred and sixty-seven; 999, nine hundred and ninety-nine.

Thousands are expressed by *four* figures; thus—2000, two thousand; 7320, seven thousand three hundred and twenty.

Tens of thousands are expressed by *five* figures; *hundreds of thousands*, by *six* figures; and so on, the numbers increasing at a tenfold rate for every additional figure.

It will thus be seen that in notation, the *rank* or place of a figure in any number is what determines the value it bears. The first figure at the *right* hand in a row of figures, always means units; the second from the right, tens; the third, hundreds; the fourth, thousands; and so on, as shewn in the following Numeration Table. Whenever a new figure is annexed at the right of a number, each of the others obtain, as it were, a promotion, and is made to express ten times its former value. Thus, 89 means 8 tens and 9 units, or eighty-nine; but if 3 be annexed, making 893, 8 means 8 hundreds, 9 means 9 tens, and 3, 3 units; or eight hundred and ninety-three. The annexing of a nothing (0) multiplies the other numbers in a similar way; thus—46, forty-six, when a nothing is annexed, becomes 460, four hundred and sixty.

The following Numeration Table shews how numbers progressively increase from units up to billions. It is read from right to left, thus—units, tens, hundreds, &c.; the rank or position in which these stand in regard to each other should be carefully studied and committed to memory :—

NUMERATION TABLE.

Billions.	Hundreds of millions.	Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
1,	9	8	7,	6	5	4,	3	2	1

The above number, 1,987,654,321, is read *one billion, nine hundred and eighty-seven millions, six hundred and fifty-four thousand, three hundred and twenty-one*. By annexing another figure, we should have *tens of billions*; another figure, *hundreds of billions*; and

another figure again, *trillions*. But notation seldom goes to such an extent; in ordinary affairs, we rarely hear of any sum beyond *billions*.

In expressing large numbers in figures, it is usual, for the sake of distinctness, to point off the figures into sets of three, by means of commas, beginning at the *right* hand, and counting towards the *left*. Thus—87,463,292.

I. To write or note down in Figures any given number.

Begin at the *left* hand, and put down the required figures one after the other, in a line, taking care to put each figure in the *place* or rank necessary to express the number, according to the Numeration Table—that is, millions must be put in the millions' place, or seventh from the right hand, and in no other; thousands in the thousands' place, or fourth from the right hand; and so on. In doing this, nothings must be put in all those *places* of which none are named in the given number. Thus, if no thousands are mentioned, a nothing must be put in the thousands' place, in order to keep the other figures in their proper rank. After the figures are written down, point them off into sets of three when necessary, beginning at the right hand.

It may be useful for the pupil first to write down as many of the places (such as units, tens, hundreds, &c.) in the Numeration Table as are required to express the given number, and then to write the respective figures below the names that express them. Thus, to write in figures, thirty-six thousand and seventy-three, first write down in a row all the places, from units up to tens of thousands, this last being the highest name in the given number; and then write the figure representing tens of thousands below that title, thousands below thousands, and so on, as follows:—

tens of thousands,	thousands,	hundreds,	tens,	units.
3	6,	0	7	3

Here there are 3 tens of thousands, 6 thousands, 0 hundreds, 7 tens, and 3 units, which, when read, are expressed as thirty-six thousand and seventy-three. It will be observed that there being no hundreds mentioned in the number, a 0 is placed in the hundreds' place to express this.

Examples.

Three hundred and forty-seven, ...	347
Twenty-six thousand four hundred and fifty-one,	26,451
Five thousand and twenty,*	5,020

*Here, as no hundreds or units are named, nothings are put down in the places of hundreds and units, in order to keep the 5 in the thousands' place, and the 2 in the tens' place. If this were not done, the number would read as 52.

Exercises.

Note down in figures the following numbers, dividing them, when necessary, by commas into sets of three:—

Seventeen, sixty-three, eighty-nine, ninety-eight, one hundred and two, one hundred and ten, one hundred and seventeen, one hundred and twenty-seven, one hundred and ninety-nine.

Two hundred, two hundred and eleven, two hundred and forty, two hundred and fifty-five, two hundred and ninety, three hundred, three hundred and eighty-eight, four hundred, four hundred and four, four hundred and seventy-six.

Five hundred, five hundred and one, five hundred and ninety-nine, six hundred, six hundred and nineteen, six hundred and thirty-seven, seven hundred, seven hundred and six, eight hundred, eight hundred and thirteen, nine hundred, nine hundred and seven, nine hundred and seventy.

One thousand, one thousand two hundred and fifty, one thousand three hundred, two thousand and forty, three thousand and four, four thousand and twenty-one, five thousand one hundred, six thousand three hundred and eleven, seven thousand and eighty-one, eight thousand nine hundred and fifteen, nine thousand nine hundred and ninety-nine.

Ten thousand, ten thousand and ten, ten thousand and fifty-nine, eleven thousand, fourteen thousand and sixteen, twenty thousand one hundred and three, thirty-three thousand and forty, sixty-four thousand and five, nine hundred and ninety-nine thousand nine hundred and ninety-nine.

One hundred thousand, two hundred thousand three hundred and eleven, seven hundred thousand and eighty, one million sixty thousand two hundred and seven, thirty-four millions one hundred and eight thousand and six, fifty millions three hundred thousand four hundred and one, eight hundred and three millions five hundred and ten thousand and ninety.

II. To read or express in Words any given number in Figures.

Begin at the *right*, and, going towards the left, name the order or rank of each figure of the given number; thus—units, tens, hundreds, thousands, &c.—that is, the first figure at the *right* is units, the next tens; and so on. Having in this way ascertained the *rank* of each figure, or its position in the Numeration Table, express the whole sum in words, reading in the usual way from *left* to right. After a little practice, it will become unnecessary to name the order or rank of the figures before reading them.

Exercises.

Read or write in words the following numbers, keeping in *mind that* the value of each figure depends on its *place* in the

Numeration Table. Thus, the first figure at the *right* always means units; the second from the right, tens; the third, hundreds; the fourth, thousands; and so on:—

13, 17, 24, 36, 49, 82, 94, 100, 110, 117, 134, 166, 199, 200, 201, 273, 219, 349, 428, 494, 511, 660, 777, 813, 979, 1,000, 1,107, 1,212, 1,347, 2,051, 3,003, 4,011, 5,100, 10,336, 20,109, 37,640, 61,420, 98,012, 100,000, 735,640, 813,105, 901,027, 2,891,563, 40,200,400, 315,070,050, 500,630,107, 850,111,005, 900,301,206.

ROMAN NOTATION.

The Romans made use of the following letters, with their combinations, to express numbers. They are still in use among ourselves for some purposes, such as the headings of chapters, divisions, &c.:—

I.	=	1	C.	=	100
V.	=	5	D. or IƆ.	=	500
X.	=	10	M. or CIƆ.	=	1000
L.	=	50			

Two or more of the *same* letter placed together, mark two or more of the same number; thus—II. means twice I., or two.

A letter of inferior value placed *before* one of superior value, means that the inferior is to be *deducted* from the superior; thus in IX., the I placed before the X means that I is to be taken from X, and IX. therefore expresses 9.

A letter of inferior value placed *after* one of superior value, means that the inferior is to be *added* to the superior—thus in LX., the X placed after L means that X is to be added to L, and LX. therefore expresses 60.

A line drawn above a letter increases its value a thousand times—as $\overline{\text{X}}$., 10,000; $\overline{\text{D}}$., 500,000.

The number IƆ (= D. or 500) is increased in value ten times for every Ɔ annexed; thus—IƆƆ. means 5000. The number CIƆ (= M. or 1000) is increased in value ten times for every C and Ɔ joined to it; thus—CIƆ., by joining C and Ɔ, becomes CCIƆƆ., or 10,000. The letters IƆ are not now in use.

I.	1	XIV.	14	LXXX.	80
II.	2	XV.	15	XC.	90
III.	3	XVI.	16	C.	100
IV.	4	XVII.	17	CC.	200
V.	5	XVIII.	18	CCC.	300
VI.	6	XIX.	19	CCCC.	400
VII.	7	XX.	20	D.	500
VIII.	8	XXI.	21	DC.	600
IX.	9	XXX.	30	DCC.	700
X.	10	XL.	40	DCCC.	800
XI.	11	L.	50	DCCCC.	900
XII.	12	LX.	60	M.	1000
XIII.	13	LXX.	70	MDCCCLIV.	1854

SIMPLE ADDITION.

Addition is the adding together of several numbers, for the purpose of finding their united amount, or what they all come to. We add or sum numbers together when we say 1 and 1 make 2; 2 and 3 make 5, &c. The amount of the numbers, when added, is called the *sum*.

Simple addition is the adding of numbers of the same kind—as, for instance, where the numbers all mean pounds, or all shillings. The rule for simple addition is given below.

Compound addition is where the numbers are partly of one kind, and partly of another—as, for instance, when some mean pounds, and some shillings. Compound addition is treated of afterwards, page 37.

The same distinction of *simple* and *compound* applies to the subsequent rules of Subtraction, Multiplication, and Division.

For the sake of saving words, certain signs are often employed in the various rules of arithmetic.

Addition is denoted by the figure of a cross, of this shape $+$. Thus, $7 + 6$ means 7 added to 6; and in order to express the sum resulting, the sign $=$, which means *equal to*, is employed, as $7 + 6 = 13$; that is, 7 and 6 are equal to 13.

ADDITION TABLE.

The following table is to be committed to memory, and should be frequently repeated forward and backward, till a readiness in adding is acquired; as—2 and 1 are 3, or 1 and 2 are 3; and so on:—

2 &	3 &	4 &	5 &	6 &	7 &	8 &	9 &
1 are 3	1 are 4	1 are 5	1 are 6	1 are 7	1 are 8	1 are 9	1 are 10
2 〃 4	2 〃 5	2 〃 6	2 〃 7	2 〃 8	2 〃 9	2 〃 10	2 〃 11
3 〃 5	3 〃 6	3 〃 7	3 〃 8	3 〃 9	3 〃 10	3 〃 11	3 〃 12
4 〃 6	4 〃 7	4 〃 8	4 〃 9	4 〃 10	4 〃 11	4 〃 12	4 〃 13
5 〃 7	5 〃 8	5 〃 9	5 〃 10	5 〃 11	5 〃 12	5 〃 13	5 〃 14
6 〃 8	6 〃 9	6 〃 10	6 〃 11	6 〃 12	6 〃 13	6 〃 14	6 〃 15
7 〃 9	7 〃 10	7 〃 11	7 〃 12	7 〃 13	7 〃 14	7 〃 15	7 〃 16
8 〃 10	8 〃 11	8 〃 12	8 〃 13	8 〃 14	8 〃 15	8 〃 16	8 〃 17
9 〃 11	9 〃 12	9 〃 13	9 〃 14	9 〃 15	9 〃 16	9 〃 17	9 〃 18
10 〃 12	10 〃 13	10 〃 14	10 〃 15	10 〃 16	10 〃 17	10 〃 18	10 〃 19

RULE FOR ADDING.

1. Write the numbers to be added under each other, figure directly under figure, in such a way that the right-hand figures will all be straight under each other, forming one even column. Thus—*units* will stand under *units*, *tens* under *tens*, *hundreds* under *hundreds*; and so on.

2. After all the numbers are thus placed, draw a line under

them; add first the *units'* column, and mark below it the right-hand figure of the amount; then carry to the *tens'* column the other figure or figures, if any (if there are none, then nothing is carried), and add it as before, including what has been brought from the units' column; put down below the *tens'* column the right-hand figure of the amount, and carry the other figures, if any, to the *hundreds'* column; add the hundreds' column, including what has been brought from the tens' column; and so on till all the columns have been added. In adding the last column, put down below it *all* the figures of the amount, as the operation is now completed.

The meaning of this operation is, that the units are first added; the units are marked down, and the tens contained in the sum carried to the next or tens' column; the tens' column is then added, the tens marked down, and the hundreds contained in the sum carried to the hundreds' column; and so on.

Example 1.—Add together 5, 3, 4, and 7.

```
 5
 3
 4
 7
——
19 Sum.
```

In this example the numbers to be added together are all single numbers or units, and, being placed under each other, they form but one column; beginning at the lowest figure of which, we add it to the one above it, the sum of the two to the next above, and so on to the top; the amount of them all is placed under the line. For example, in the present account, we begin by saying 7 and 4 are 11, 11 and 3 are 14, 14 and 5 are 19, which is then placed under the line.

Example 2.—Add together 27, 5, 536, 352, and 275.

```
  27
   5
 536
 352
 275
————
1195
```

In this example, beginning at the lowest figure of the right-hand column, we say 5 and 2 are 7, and 6 are 13, and 5 are 18, and 7 are 25—that is, 2 tens and 5 units; but since there are other columns to be added, we put down the 5 only, under the units' column, and *carry* or add the 2 tens to the lowest figure of the next column, saying 2 and 7 are 9, and 5 are 14, and 3 are 17, and 2 are 19. Here, as before, the 9 only is put down under the second column, and the 1 *carried* to the next; thus—1 and 2 are 3, and 3 are 6, and 5 are 11. No more figures remaining to be added, both these figures are now put down, and the amount or *sum* of them all is found to be 1195.

If the amount of any column be in three figures, still only the last or right-hand figure of the three is to be put down, and the other two *carried* to the next column. For example, if the amount of a column be 127, put down the 7 and carry the other two—viz. 12; if it be 234, put down the 4 and carry 23. Thus, all the figures except the last or right-hand one are to be carried to the succeeding column.

PROOF.—To prove that any question in addition is correctly wrought, we may add from top to bottom of the columns, and see if the sum be the same as by adding from bottom to top. Another method consists in striking off the upper row of figures, making

the addition without them, and then adding the top row to the sum. For example, treat the above question as follows:—

27
5
536
352
275
1168
27
1195

Here the top row, 27, is struck off, and being afterwards added to the product, the result is the same as if all had been added up together; this proves that the summation was properly performed.

Advice to Learners.—The art of adding up quickly is considered a great accomplishment, and we strongly recommend young persons to acquire it with all reasonable diligence. It will save much time if they learn to sum up the columns by a glance of the eye, without naming the numbers; for instance, taking the above question, instead of saying 5 and 2 are 7, 7 and 6 are 13, 13 and 5 are 18, 18 and 7 are 25, *acquire the knack of summing up the figures in the mind; thus*—5, 7, 13, 18, 25.

Exercises.

1.	2.	3.	4.	5.	6.	7.	8.	9.
3	6	16	27	55	317	450	412	653
2	7	13	30	61	68	73	643	705
1	9	10	43	18	40	307	507	690
4	0	24	78	70	7	64	618	437
2	5	32	65	49	5	176	370	869

10. George has 13 marbles, John has 19, James 16, William 20, and Thomas 27. How many have they in all? . . Ans. 95

11. Charles got several bundles of pens to count: he found in one bundle 24, in another 37, in another 53, in another 40, and in the last 17. How many were there in all? . . Ans. 171

12. Add together 86, 5, 41, 7, 26, and 357, . . . " 522

13. Add 45, 60, 764, 37, and 78, ... " 984

14. Add 375, 460, 85, 67, and 43, ... " 1030

15. Add 4763, 2768, 437, and 8326, ... " 16294

16. How many do 735, 4628, 39, and 57 come to? . " 5459

17. How many are 85, 79, 632, and 781? . . " 1577

18.	19.	20.	21.	22.	23.
764	565	650	5780	6038	5634
30	436	392	36	47	7325
7	398	415	541	651	608
69	325	686	6837	8306	4620
307	791	735	906	8	76
240	452	606	78	97	8

Answers.

1. 12	4. 248	7. 1070	18. 1417	21. 14178
2. 27	5. 253	8. 2550	19. 2967	22. 15147
3. 95	6. 437	9. 3354	20. 3484	23. 18266

24.	25.	26.	27.	28.	29.
8705	9837	8521	7235	9167	5678
6817	7754	3947	5984	3582	9012
596	6383	1876	1678	9764	3456
7384	3847	2345	2959	1852	7891
769	2905	9812	3827	3956	2345
93	9978	7195	1893	1234	6789

30.	31.	32.	33.	34.	35.
3957	7854	3954	3198	9735	9162
1268	3141	1276	4569	1976	3976
9854	6957	3919	7352	3759	8537
7369	2718	4578	1127	4169	7852
1275	2845	9037	3964	3785	3964
9631	9637	1685	3576	2135	8572

36.	37.	38.	39.	40.	41.
6783	4068	8964	4065	4209	5126
2954	2379	4253	2387	5670	9632
3926	9965	7068	4021	8463	7923
1483	8470	3829	5689	2338	8034
1846	2386	4956	9148	4075	9146
3972	9742	9748	2796	4687	1752

42.	43.	44.	45.	46.	47.
2677	6973	2932	1352	5176	5310
8821	2185	7812	3976	3498	6781
9678	9657	9637	4757	5132	9574
5822	8326	1285	3182	6790	3448
9172	9577	4037	7957	9259	5486
6854	8763	1877	2632	3807	4798

48.	49.	50.	51.	52.	53.
6237	3789	7984	3043	2463	6287
9743	9415	3196	8923	4087	4509
8034	9789	9768	9748	5868	6243
9154	6431	8437	2396	4293	7801
9257	9283	1689	5148	8068	9360
2863	7965	9874	2981	3647	4918

Answers.

24. 24364	30. 33354	36. 20964	42. 42024	48. 45288
25. 40704	31. 33152	37. 37010	43. 45481	49. 46672
26. 33696	32. 24449	38. 38818	44. 27580	50. 40948
27. 23571	33. 23786	39. 28106	45. 23856	51. 32239
28. 29555	34. 25559	40. 29442	46. 33662	52. 28426
29. 35171	35. 42063	41. 41613	47. 35397	53. 39118

54.	55.	56.	57.	58.	59.
7817	1297	59317	37634	76305	8476
7939	9084	98765	58427	3756	6857
2157	3275	39278	35376	48	473
9736	7926	41692	86432	764	64
5273	1837	54183	47680	87635	5796
1892	2749	27952	70637	5762	7032
3685	8475	61847	64075	67053	684

60.	61.	62.	63.	64.
91934	473926	397426	825927	39257
89756	593842	593075	259728	83491
19372	795384	329705	298732	78251
49108	682173	574839	987659	23567
21685	373925	328975	283139	90123
39724	814198	895763	173916	57925
58372	765432	654321	876544	39587
27998	127904	718902	328597	78276
54796	752847	537198	123456	39718

65. What is the amount of seventeen thousand three hundre and ten, five hundred and seven, two thousand four hundred an fifty, and fifty thousand one hundred and twelve? . Ans. 7037

66. Add together four thousand two hundred and eleven, tw thousand and forty, six hundred and twenty-seven, ninety-eigh and seven thousand nine hundred and three, . . Ans. 1487

Answers.

54. 38499	57. 400261	60. 452745	63. 415769
55. 34643	58. 241328	61. 5379631	64. 530199
56. 383034	59. 293852	62. 5030204	

SIMPLE SUBTRACTION.

Subtraction is the taking or deducting of a smaller numbe from a greater, to find what remains, or what is the differenc between them. The number left after deducting the one from th other, is called the *remainder*.

We subtract when we say, 3 from 5, and 2 remains. Here 2 i the difference between 3 and 5. If John has 5 marbles, and h gives James 3 of them, he will have 2 remaining.

Subtraction is denoted by a small horizontal line, thus [—] between two figures; as, for example, $9 - 5 = 4$, means, 5 subtracted from 9, and 4 remains.

SUBTRACTION TABLE.

2 from	3 from	4 from	5 from	6 from	7 from	8 from	9 from
3 − 1	4 − 1	5 − 1	6 − 1	7 − 1	8 − 1	9 − 1	10 − 1
4 ″ 2	5 ″ 2	6 ″ 2	7 ″ 2	8 ″ 2	9 ″ 2	10 ″ 2	11 ″ 2
5 ″ 3	6 ″ 3	7 ″ 3	8 ″ 3	9 ″ 3	10 ″ 3	11 ″ 3	12 ″ 3
6 ″ 4	7 ″ 4	8 ″ 4	9 ″ 4	10 ″ 4	11 ″ 4	12 ″ 4	13 ″ 4
7 ″ 5	8 ″ 5	9 ″ 5	10 ″ 5	11 ″ 5	12 ″ 5	13 ″ 5	14 ″ 5
8 ″ 6	9 ″ 6	10 ″ 6	11 ″ 6	12 ″ 6	13 ″ 6	14 ″ 6	15 ″ 6
9 ″ 7	10 ″ 7	11 ″ 7	12 ″ 7	13 ″ 7	14 ″ 7	15 ″ 7	16 ″ 7
10 ″ 8	11 ″ 8	12 ″ 8	13 ″ 8	14 ″ 8	15 ″ 8	16 ″ 8	17 ″ 8
11 ″ 9	12 ″ 9	13 ″ 9	14 ″ 9	15 ″ 9	16 ″ 9	17 ″ 9	18 ″ 9
12 ″ 10	13 ″ 10	14 ″ 10	15 ″ 10	16 ″ 10	17 ″ 10	18 ″ 10	19 ″ 10

RULE FOR SUBTRACTING.

1. Place the smaller number below the greater, writing units under units, tens under tens, and so on, as in Addition.

2. Draw a line under them, and beginning at the *right* hand, deduct in succession each figure in the lower line from the figure immediately above it in the upper line, marking down the remainder below each figure.

3. If any figure in the lower line be greater than the figure above it, add 10 to the upper figure, and then go on with the subtraction. The 10 thus added is said to be borrowed from the next upper figure, and, as an equivalent for having borrowed it, the next *under* figure requires to be considered as 1 *more* before subtracting.

The reason that the 10 borrowed from the next figure is counted as 1 in making an equivalent allowance for it, is, that the figure from which 10 is borrowed is of a higher order or rank than that to which it is carried; and, consequently, 1 of the former is equal to 10 of the latter. The 10 borrowed is allowed for by making the next *under* figure 1 more, instead of making the next *upper* figure 1 less (as, strictly speaking, should be done), because it is more convenient in practice, and produces the same result.

Example 1.—Take 325 from 537.

$$\begin{array}{r} 537 \\ 325 \\ \hline 212 \end{array}$$ *Remainder.*

Here, 325 being the smaller number, it is placed under 537, the greater, and commencing at 5, we take 5 from 7, and the remainder is 2, which we place below the line, directly under the 5; then proceeding to the next figure, we say 2 from 3, and 1 remains, which is also placed under the line; then going on to the next figure, we say 3 from 5, and 2 remains; this being placed below the line, the whole remainder or difference between the two numbers is found to be 212.

Example 2.—From 65074 take 4054.

65074
4054
61020

Commencing as before, at the right-hand figure, 4, we find the figure immediately above it is 4 also; and when 4 is taken from 4, nothing remains; a nothing is therefore placed under the line: passing to the next figure, we say 5 from 7, and 2 remains; the next figure being a nothing, and the one above it also a nothing, nothing remains; this being placed under the line, we pass to the next figure, and say 4 from 5, and 1 remains; passing to the next, we find no figure under the 6; there is therefore nothing to be taken from it, and 6 is put down under the line; the whole remainder is found to be 61020.

These examples, it will be found, are very simple, because all the under figures of the numbers are either less or exactly equal to the figures immediately above them; but it often happens that some of the under figures are greater than those just above them. We have, therefore, to shew what is to be done in that case.

Example 3.—From 8432 take 6815.

8432
6815
1617

Here, it will be observed, that 5, the figure with which we begin, stands to be subtracted from 2; but 5 cannot be taken from 2, because 5 is a greater number than 2. We add, therefore, 10 to the 2, making it 12, and then say 5 from 12, and 7 remain. Now, because 10 was added to the 2, we are said to have borrowed *one* from the figure just before it, which *one* is to be repaid by carrying, or adding 1 to the next *under* figure we come to. Passing, therefore, to the next figure in the above example, we add 1 to the 1 making 2, and 2 from 3, the upper figure, 1 remains; next we find 8 from 4; but as we cannot take 8 from 4, we add 10 to it, making it 14, and we say 8 from 14, and 6 remain; and here, again, having *borrowed one*, we carry it, as before, to the next under figure 6, making 7, and 7 from 8, 1 remains.

PROOF.—To prove that any question in subtraction is correctly wrought, add the *remainder* to the lower row of figures, and if their sum be equal to the upper row, the account is correct.

For example, treat the foregoing question as follows:—

8432
6815
1617
8432

Here it is seen that, by adding the remainder, 1617, to the lower row, 6815, we bring out 8432, or the upper row, as the result or sum—the account is, consequently, correct. Subtraction, therefore, may always be proved by addition.

Exercises.

1.	2.	3.	4.
7348	96803	57462	47630
5234	63801	3421	5420

Answers.

1. 2114	2. 33002	3. 54041	4. 42210

Exercises.

5.	6.	7.	8.	9.
63247	72068	570346	786923	853065
45164	51739	82182	92875	607043

10.	11.	12.	13.	14.
719384	594768	938726	839269	947385
206123	123456	124311	627135	413052

15.	16.	17.	18.
543912647	835164295	594168231	615932718
275183176	187106837	427641897	309168271

19.	20.	21.	22.
184391715	315712937	791936752	683175398
139276189	182619308	139271825	596483712

23. From 50602 take 32476, ... Ans. 18126
24. From 173056 take 92674, ... " 80382
25. Subtract 14320 from 32476, ... " 18156
26. Subtract 7560 from 13463, . . . " 5903
27. What is the difference between 37651 and 28430? " 9221
28. What is the difference between 7326 and 9718? " 2392
29. How much is 3681 less than 5340? . . . " 1659
30. How much is 53160 more than 2687? . " 50473

31. A woman went to market with 346 eggs in her basket; she sold 288. How many did she return with? . . . Ans. 58

32. From a piece of cloth consisting of 153 yards, a merchant sold 27 yards to A, and 48 yards to B. How much of the piece remained? . . . . Ans. 78

33. Lucy went to make some purchases with 34 shillings in her purse. She bought a pound of tea for 4 shillings, 4 pounds of sugar for 2 shillings, 6 pounds of soap for 3 shillings, and 6 yards of linen for 14 shillings. How much should she have over?
Ans. 11s.

34. America was discovered in 1492. How long is it since?
Ans. (in 1854) 362

35. What age now is a person who was born in 1803?
Ans. (in 1854) 51

Answers.

5. 18083	9. 246022	13. 212134	17. 166526334
6. 20329	10. 513261	14. 534333	18. 306764447
7. 488164	11. 471312	15. 268729471	19. 45115526
8. 694048	12. 814415	16. 648057458	20. 133093629
	21. 652664927	22. 86691686	

SIMPLE MULTIPLICATION.

Multiplication is the method of ascertaining what a given number will amount to, when repeated a certain number of times; as, for example, what 6 repeated 4 times will amount to; or, in other words, how many are 4 times 6?

We know how this is to be done by Addition: the figures are placed under each other, and we say, 6 and 6 are 12, and 6 are 18, and 6 are 24. In Multiplication, however, a different plan is adopted: the 6 and 4 are said to be multiplied together—that is, we multiply 6 by 4, and say at once, 4 times 6 are 24. It is usually most convenient in multiplying two numbers together, to multiply the larger number by the smaller.

The number to be multiplied is called the *multiplicand;* the number that multiplies it, the *multiplier;* and the result of multiplying the two numbers together, the *product.*

A number that is produced by the multiplication of two other numbers, is called a *composite* number—as, for instance, 30, which is the product of 5 and 6. The 5 and 6, are called the *factors* of 30, and 30 is also said to be a multiple of either of these numbers.

A number which cannot be produced by the multiplication of two other numbers, is called a *prime number.*

Multiplication is denoted by a cross of this shape ×; thus, $3 \times 8 = 24$, signifies, that by multiplying 8 by 3, the product is 24.

The process of multiplication is carried on by means of the following Multiplication Table, which shews how much certain numbers amount to, when multiplied together.

The table should be carefully committed to memory, as a knowledge of it is of great value in arithmetic, and saves much trouble in after-life.

MULTIPLICATION TABLE.

2 times	3 times	4 times	5 times	6 times	7 times	8 times	9 times	10 times	11 times	12 times
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7	1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 ″ 4	2 ″ 6	2 ″ 8	2 ″ 10	2 ″ 12	2 ″ 14	2 ″ 16	2 ″ 18	2 ″ 20	2 ″ 22	2 ″ 24
3 ″ 6	3 ″ 9	3 ″ 12	3 ″ 15	3 ″ 18	3 ″ 21	3 ″ 24	3 ″ 27	3 ″ 30	3 ″ 33	3 ″ 36
4 ″ 8	4 ″ 12	4 ″ 16	4 ″ 20	4 ″ 24	4 ″ 28	4 ″ 32	4 ″ 36	4 ″ 40	4 ″ 44	4 ″ 48
5 ″ 10	5 ″ 15	5 ″ 20	5 ″ 25	5 ″ 30	5 ″ 35	5 ″ 40	5 ″ 45	5 ″ 50	5 ″ 55	5 ″ 60
6 ″ 12	6 ″ 18	6 ″ 24	6 ″ 30	6 ″ 36	6 ″ 42	6 ″ 48	6 ″ 54	6 ″ 60	6 ″ 66	6 ″ 72
7 ″ 14	7 ″ 21	7 ″ 28	7 ″ 35	7 ″ 42	7 ″ 49	7 ″ 56	7 ″ 63	7 ″ 70	7 ″ 77	7 ″ 84
8 ″ 16	8 ″ 24	8 ″ 32	8 ″ 40	8 ″ 48	8 ″ 56	8 ″ 64	8 ″ 72	8 ″ 80	8 ″ 88	8 ″ 96
9 ″ 18	9 ″ 27	9 ″ 36	9 ″ 45	9 ″ 54	9 ″ 63	9 ″ 72	9 ″ 81	9 ″ 90	9 ″ 99	9 ″ 108
10 ″ 20	10 ″ 30	10 ″ 40	10 ″ 50	10 ″ 60	10 ″ 70	10 ″ 80	10 ″ 90	10 ″ 100	10 ″ 110	10 ″ 120
11 ″ 22	11 ″ 33	11 ″ 44	11 ″ 55	11 ″ 66	11 ″ 77	11 ″ 88	11 ″ 99	11 ″ 110	11 ″ 121	11 ″ 132
12 ″ 24	12 ″ 36	12 ″ 48	12 ″ 60	12 ″ 72	12 ″ 84	12 ″ 96	12 ″ 108	12 ″ 120	12 ″ 132	12 ″ 144

The following is a more condensed form of the Multiplication Table. The calculations are carried as far as 20 times 20 :—

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

When any number in the top row of the table is multiplied by any number in the left-hand *side* row, the product is found in the compartment or square *beneath* the former, and *opposite* the latter. Thus —2 times 2 are 4; 5 times 6 are 30; 20 times 20 are 400.

RULES FOR MULTIPLICATION.

I. When the Multiplier does not exceed 12.

Rule.—Write down the number to be multiplied; place the multiplier below it, at the right-hand side, and draw a line under them; then begin at the right hand, and multiply each successive figure in the multiplicand, by the multiplier; mark down the units of each product below the figure multiplied, and carry the *tens*, as in Addition, to the next figure, when it is multiplied in its turn. When all the figures have been multiplied, the result is the answer required.

Example.—Multiply 27 by 9.

27	*Multiplicand.*
9	*Multiplier.*
243	*Product.*

Here, beginning with the right-hand figure, we say 9 times 7 are 63; putting down 3, we carry 6 to the next figure, and say, 9 times 2 are 18, and 6 which was carried, make 24; and writing down both of these figures, as there are no more to multiply, the product is found to be 243.

In Multiplication, the *tens* are carried, and the remainders marked down at each stage of the process, for the same reasons as are explained in Simple Addition, page 7.

Exercises.

1.	Multiply	75683	by	2, 3, 4, 5.
2.	"	986054	"	6, 7, 8, 9.
3.	"	6810796	"	5, 7, 8, 9.
4.	"	1852963074	"	2, 3, 4, 6, 7, 8, 9, 12.
5.	"	6837014925	"	2, 3, 4, 5, 7, 8, 9, 12.

II. When the Multiplier exceeds 12.

Rule.—Write the multiplier below the number to be multiplied, placing units under units, tens under tens, and so on. Then, beginning at the right, multiply the number by each figure of the multiplier in succession, placing each new line of products below the previous one, but a *place* further to the left; so that each line may commence exactly below the figure in the multiplier producing it: when a nothing occurs in the multiplier, pass on to the next figure. Then add up all the lines of products, and their sum is the product required.

Examples.—Multiply 5463 by 34; and 76843 by 4563.

1.

```
   5463
     34
  -----
  21852
 16389
 ------
 185742 Ans.
```

2.

```
     76843
      4563
    ------
    230529
   461058
  384215
 307372
 ---------
 350634609 Ans.
```

In example 1, the number is multiplied first by the 4, the product of which being written down, we proceed to multiply by the 3, and place its product below the other, but one place further to the left, the first figure being put immediately below the 3. A line is then drawn under them, and they are added together.

The reason for writing each successive line of products, a place further to the left, as in the above examples of Multiplication, is, that each figure of the multiplier, counting from *right* to *left*, is a place higher in order than the one next to it, and, therefore, in multiplying, each new product will be of a higher order than the one preceding it, and must accordingly be written one place further to the left.

Answers.

1.	151366	227049	302732	378415
2.	5916324	6902378	7888432	8874486
3.	34053980	47675572	54486368	61297164
4.	3705926148	5558889222	7411852296	11117778444
	12970741518	14823704592	16676667666	22235556888
5.	13674029850	20511044775	27348059700	34185074625
	47859104475	54696119400	61533134325	82044179100

Thus, to multiply 436 by 324 according to the rule, is the same as to multiply separately by 4 units, 2 tens, and 3 hundreds—

```
   1.                          2.

   436        436 ×   4 =   1744
   324            ×  20 =   8720
  ----            × 300 = 130800
  1744                    ------
  872                     141264
 1308
 ------
 141264
```

Here it will be seen that the result of multiplying is the same in both cases. In No. 2, the figures have their proper position given to them, according to the rank of the multiplier, by means of nothings annexed, and in No. 1 by putting each line of figures a place further to the left, which serves the same purpose as the nothings.

PROOF.—To prove that any question in multiplication is correctly performed, we may reverse the operation, by making the multiplicand the multiplier; and if the product is the same, the account is correct.

```
 36        27
 27        36
---       ---
252       162
72        81
---       ---
972       972
```

For example, if we multiply 36 by 27, the product is 972; and if we multiply 27 by 36, the product will also be 972.

Exercises.

Multiply the following numbers :—

				Answers.	
1.	18530729	by 21	42	389145309	778290618
2.	43915806	〃 37	26	1624884822	1141810956
3.	70268315	〃 51	84	3583684065	5902538460
4.	92573684	〃 78	89	7220747352	8239057876
5.	39753984	〃 47	96	1868437248	3816382464
6.	48637251	〃 73	67	3550519323	3258695817
7.	83974695	〃 89	23	7473747855	1931417985
8.	18370298	〃 57	98	1047106986	1800289204
9.	95721386	〃 79	89	7561989494	8519203354
10.	84397857	〃 75	605	6329839275	51060703485
11.	39260489	〃 38	406	1491898582	15939758534
12.	17935982	〃 207	98	3712748274	1757726236
13.	31694708	〃 78	59	2472187224	1869987772
14.	89357064	〃 65	91	5808209160	8131492824
15.	39712584	〃 456	789	18108938304	31333228776
16.	80379218	〃 372	958	29901069096	77003290844
17.	92873905	〃 837	325	77735458485	30184019125
18.	61938796	〃 504	982	31217153184	60823897672
19.	54917283	〃 271	659	14882583693	36190489497
20.	93650389	〃 948	326	88780568772	30530026814

Multiply—				Answers.	
21.	41705826 by	516	486	21520206216	20269031436
22.	18472593 〃	785	827	14500985505	15276834411
23.	85149260 〃	6521	61594	555258324460	5244683520440
24.	52816937 〃	3298	73261	174190258226	3869421621557
25.	29583604 〃	19695	4938	582649080780	146083836552
26.	96250371 〃	86372	8476	8313337044012	815818144596
27.	63927048 〃	9328	60745	596311503744	3883248530760
28.	30694715 〃	8975	32164	275485067125	987264813260

29. Two factors are 30957 and 839; what is their product?
Ans. 25972923

30. How many oranges are there in 47 boxes, when each box contains 279? . . . . Ans. 13113

31. In a desk there were 6 drawers, each drawer was divided into 8 compartments, and in each compartment were 87 pounds; how many pounds did the desk contain? . . . Ans. 4176

III. When the Multiplier is a number having *nothings* annexed to it.

Rule.—Extend the nothings beyond the multiplicand, then write them down as part of the answer, and multiply by the other figure or figures of the multiplier, as in Rules I. and II.

Examples.—Multiply 7348 by 70; 47683 by 70600; and 87300 by 760.

1.	2.	3.
7348	47683	87300
70	70600	760
514360 *Answer.*	28609800	5238000
	333781	611100
	3366419800 *Answer.*	66348000 *Answer.*

Exercises.

				Answers.
1. Multiply	3179408	by	20	63588160
2. 〃	87039	〃	6090	530067510
3. 〃	930675	〃	8700	8096872500
4. 〃	7856973	〃	590600	4640328253800
5. 〃	19076573	〃	80090	1527842731570

Note.—When the multiplier is 10, 100, 1000, or 1 with any other number of nothings annexed, the multiplication is accomplished merely by annexing as many nothings to the multiplicand as are contained in the multiplier; thus—

386 is multiplied by 10, by annexing 1 nothing = 3860
736 〃 〃 1000, 〃 〃 3 nothings = 736000

IV. When the Multiplier is a Fraction—as $\frac{1}{2}$, or $\frac{3}{4}$.*

Rule.—Multiply the given number by the upper figure of the fraction, and divide† the product by the under figure.

To multiply a number by a fraction, such as $\frac{1}{2}$, $\frac{3}{4}$, &c., is to find what is the *half* or *three-fourths* of the number: thus—to multiply 16 by $\frac{3}{4}$, is the same thing as to find how much is $\frac{3}{4}$ of 16.

† For the process of *division*, see page 20.

Example.—Multiply 16 by $\frac{3}{4}$.

$$\begin{array}{r} 16 \\ \underline{3} \\ 4\overline{)48} \\ \overline{12} \end{array}$$
Answer.

Here 16 is multiplied by 3 and then divided by 4.

When the upper figure of a fraction is 1, such as $\frac{1}{3}$, it is unnecessary to multiply by the 1, as it will obviously make no difference on the number; all that is required is to divide the given number by the *under* figure; thus—to multiply 24 by $\frac{1}{3}$, we divide 24 by 3, and the answer is 8—that is, 8 is one-third of 24.

V. When the Multiplier is an integer or *whole* number, with a Fraction annexed—as $8\frac{2}{3}$.

Rule.—Multiply first by the integer, then by the fraction as in Rule IV., and add both the products for the answer.

Example.—Multiply 4387 by $7\frac{5}{6}$.

$$\begin{array}{r} 4387 \times 7 \\ \underline{7} \\ 30709 \\ \underline{3655\frac{5}{6}} \\ 34364\frac{5}{6} \end{array}$$
Answer.

Here $3655\frac{5}{6}$, the product of multiplying by $\frac{5}{6}$, is added to the product of multiplying by 7.

$$\begin{array}{r} 4387 \times \frac{5}{6} \\ \underline{5} \\ 6\overline{)21935} \\ \overline{3655\frac{5}{6}} \end{array}$$

Exercises.

1. Multiply	5376	by	$5\frac{1}{2}$
2. "	7493	"	$6\frac{1}{4}$
3. "	8375	"	$7\frac{2}{3}$
4. "	17654	"	$8\frac{4}{5}$
5. Multiply	89705	by	$9\frac{7}{8}$
6. "	73476	"	$15\frac{3}{7}$
7. "	89596	"	$85\frac{7}{12}$
8. "	73685	"	$376\frac{8}{9}$

Answers.

1. 29568
2. $46831\frac{1}{4}$
3. $64208\frac{1}{3}$
4. $155355\frac{1}{5}$
5. $885836\frac{7}{8}$
6. $1133629\frac{5}{7}$
7. $7667924\frac{4}{12}$
8. $27771057\frac{7}{9}$

* See explanation of Fractions, under that head, p. 67.

SIMPLE DIVISION.

DIVISION is that process by which we discover how often one number is contained in another, or by which we divide a given number into any proposed number of equal parts. We can ascertain, by aid of the Multiplication Table, how many times any number is contained in another, as far as 144, or 12 times 12, without writing figures; but the calculations beyond this point are usually written down.

The number to be divided, is called the *dividend;* the number by which it is divided, is the *divisor;* and the result is termed the *quotient*, from a Latin word, which means, literally, How many times?

Division is denoted by the following character $\div$; thus—75 $\div$ 25, signifies that 75 is to be divided by 25.

RULES FOR DIVISION.

I. WHEN THE DIVISOR DOES NOT EXCEED 12.

RULE.—1. Write down the dividend; draw a curved line on the left side of it, and a straight line below it: then write the divisor on the left of the curved line.

2. Find how often the divisor is contained in the first figure, or (if the divisor is larger than it) in the *two* first figures of the dividend, and write the quotient below.

3. Multiply the divisor by the quotient, and deduct the product from the figure or figures just divided; then annex to the remainder, if any, the next figure of the dividend, find how often the divisor is contained in this new sum,* and write down the quotient; and so on as before, till all the figures of the dividend have been divided, when the division is completed.

If there is a remainder after the division is finished, it is marked down as part of the answer, with the divisor written below it, forming a fraction.

* NOTE.—If, after annexing a figure to the remainder, the number be less than the divisor, place a *nothing* in the quotient to express this, then annex another figure to the remainder, and proceed with the division.

The process of division here described, is termed *Short Division* when part of the process is carried on in the mind, and the results only written down: short division is employed when the divisor does not exceed 12. In numbers above 12, it is necessary, for convenience, to write down at length the various steps of the process; and when this is done, it is termed *Long Division.* The *principle* is the same in both cases, the sole difference being, that in the one, the operation is only partly written down, whilst in the other, all the figures of the process

are written; as will be seen in the following example, in which both methods are given :—

Example.—Divide 7958 by 6.

Short Division.

$$\begin{array}{l} 6\underline{)7958} \\ \quad 1326\frac{2}{6} \textit{ Quot.} \end{array}$$

Long Division.

$$\begin{array}{l} 6)7958(1326\frac{2}{6} \textit{ Quot.} \\ \underline{6} \\ 19 \\ \underline{18} \\ \;\;15 \\ \;\;\underline{12} \\ \;\;\;\;38 \\ \;\;\;\;\underline{36} \\ \;\;\;\;\;\;2 = \frac{2}{6} \end{array}$$

Here we find that there is *one* 6 in 7, the first figure of the dividend, and 1 over; we therefore write 1 in the quotient, and multiplying the divisor, 6, by 1, the quotient, subtract the product, 6, from 7 of the dividend. To the remainder, 1, we bring down 9, the next figure of the dividend, making 19. As there are 3 times 6 in 19, we place 3 in the quotient, and multiplying the divisor, 6, by 3, subtract 18 from 19, which leaves 1. To this 1 we bring down 5, making 15; and as there are 2 times 6 in 15, we place 2 in the quotient, and multiplying 6 by 2, subtract 12 from 15, leaving 3. To this 3 we bring down 8, making 38, in which there are 6 sixes; therefore, placing 6 in the quotient, we multiply 6 by 6, and subtract 36 from 38, leaving 2 over. Here the account terminates, it being found that there are 1326 sixes in 7958, with a remainder of 2, below which the divisor is written, thus—$\frac{2}{6}$, and the fraction is annexed to the quotient as part of the answer.

Exercises.

Divide the following numbers :—

						Answers.		
1.	398654	by 2	3	4		199327	$132884\frac{2}{3}$	$99663\frac{2}{4}$
2.	87564329	〃 5	6	7		$17512865\frac{4}{5}$	$14594054\frac{5}{6}$	$12509189\frac{6}{7}$
3.	876437639	〃 2	4	8		$438218819\frac{1}{2}$	$219109409\frac{3}{4}$	$109554704\frac{7}{8}$
4.	〃	〃 9	11	12		$97381959\frac{8}{9}$	79676149	$73036469\frac{11}{12}$

II. When the Divisor exceeds 12.

Rule.—1. Draw a curved line on each side of the dividend, and place the divisor on the left of it.

2. Point off as many figures from the left of the dividend as make a number greater than the divisor: find how often the divisor is contained in this number, and place the result at the *right* side of the dividend, as the first figure of the quotient. It is to be observed, that 9 is the highest number to be placed in the quotient at any one time.

3. Multiply the divisor by the quotient, and subtract the product from the number pointed off; then bring down and annex to the remainder, if any, the next figure of the dividend.* Proceed

* Note.—If, after bringing down a figure to any of the remainders, during the process, the number be less than the divisor, place a nothing in the quotient to express this, then bring down another figure to the remainder, and proceed with the division.

with this new sum as before, placing the quotient as the next figure of the answer; and so on till all the figures of the dividend have been brought down and divided, when the operation is completed. If there is a remainder after the division is finished, it is marked down as part of the answer, with the divisor written below it, forming a fraction.

It will be found useful, on bringing down in succession the figures to the remainders, to place dots below them in the dividend, to prevent any mistakes as to what have been brought down.

When the divisor is large, and it cannot be seen at a glance what the quotient figure in any case should be, it will usually be a guide to the required figure (except when it is 1) to take as a *trial* figure the number of times that the first figure of the divisor is contained in the *first* figure, or first two figures of the dividend. This will give an approximation, and will help to shew what the true figure should be.

Example.—Divide 494033 by 239.

239)494033(2067 $\frac{20}{239}$
478
1603
1434
1693
1673
20 = $\frac{20}{239}$

In this example, finding that there are two times 239 in 494, we place 2 in the quotient; then multiply 239 by 2, making 478, which we subtract from 494. To the remainder, 16, we bring down 0, the next figure in the dividend, making 160; and, finding that 239 is not contained in 160, we place a 0 in the quotient, and bring down 3, the next figure in the dividend, making 1603. In this sum, 239 being contained 6 times, we place 6 in the quotient, and multiply 239 by 6, making 1434, which we subtract from 1603. To the remainder, 169, we bring down 3, the last figure in the dividend, making 1693, in which 239 is contained 7 times; we place 7 in the quotient, and multiply 239 by 7, making 1673, which, being deducted from 1693, leaves a remainder of 20, written, with the divisor below it, as $\frac{20}{239}$.

THE REASON for the rule of Division will appear from the following example:—

Divide 94608 by 73.

1.

73)94608(1000
73000
21608(200
14600
7008(90
6570
438(6
438
Total, 1296 *times*.

2.

73)94608(1296
73...
216
146
700
657
438
438

Here it will be seen, on comparing No. 1 with No. 2, that the process is virtually the same as if we were first to ascertain how many *thousand* times 73 is contained in the dividend; how many *hundred* of times in the remainder; how many times *ten* in the next remainder; and how many times *one* in the last remainder; and adding all these, we

have the total number of times. In this example, we have 73 contained in the dividend and the successive remainders, 1000, 200, 90, and 6 times—in all, 1296 times. It will be seen, that the nothings annexed to the quotients, &c., in No. 1, may, in practice, be left out, as the other figures have the same value without them, by being placed as in No. 2, according to their rank.

Proof.—To prove that any question in division is correctly performed, multiply the quotient by the divisor, and add to the product any remainder. The answer will be the same as the dividend, if the working has been correct.

Example.—Divide 4966 by 37.

```
37)4966(134
   37
   ---
   126
   111
   ---
    156
    148
    ---
      8
```

Here, on dividing 4966 by 37, the answer is 134, and a remainder of 8; and on multiplying the quotient by the divisor, and adding the remainder, the product is the same as the dividend.

```
Proof.
 134⁸
  37
 ---
 938
402
   8 Remainder.
----
4966
```

Exercises.

Divide the following numbers :—

1. 79512587 by	13	23	31		9. 12345678 by	68	79
2. 89659053 "	34	41	73		10. 90273189 "	97	432
3. 19271873 "	51	43	83		11. 87625432 "	199	843
4. 85296307 "	61	71	85		12. 17927618 "	925	379
5. 41824680 "	62	52	74		13. 419352716 "	123	456
6. 36925814 "	82	53	65		14. 900416824 "	54321	98765
7. 70369257 "	91	17	19		15. 519387549 "	17297	2731
8. 84169273 "	29	37	38		16. 183926157 "	37246	8799

Answers.

1.	5.	9.	13.
$6116352\frac{11}{13}$	$674591\frac{38}{62}$	$181554\frac{6}{68}$	$3409371\frac{83}{123}$
3457069	$804320\frac{40}{52}$	$156274\frac{32}{79}$	$919633\frac{68}{456}$
$2564922\frac{5}{31}$	$565198\frac{28}{74}$		
2.	**6.**	**10.**	**14.**
$2637030\frac{33}{34}$	$450314\frac{66}{82}$	$930651\frac{42}{97}$	$16575\frac{46249}{54321}$
$2186806\frac{7}{41}$	$696713\frac{25}{53}$	$208965\frac{309}{432}$	$9116\frac{75084}{98765}$
$1228206\frac{15}{73}$	$568089\frac{29}{65}$		
3.	**7.**	**11.**	**15.**
$377879\frac{44}{51}$	$773288\frac{49}{91}$	$440328\frac{160}{199}$	$30027\frac{10530}{17297}$
$448183\frac{4}{43}$	$4139368\frac{1}{17}$	$103944\frac{640}{843}$	$190182\frac{507}{2731}$
$232191\frac{20}{83}$	$3703645\frac{2}{19}$		
4.	**8.**	**12.**	**16.**
$1398300\frac{7}{61}$	$2902388\frac{21}{29}$	$19381\frac{193}{925}$	$4938\frac{5409}{37246}$
$1201356\frac{31}{71}$	$2274845\frac{8}{37}$	$47302\frac{160}{379}$	$20903\frac{660}{8799}$
$1003485\frac{82}{85}$	$2214980\frac{33}{38}$		

NOTE.—WHEN THE DIVISOR is the product of two numbers, neither of which exceeds 12, *short* division may be employed, by dividing the dividend by one of the numbers, and the quotient of this by the other.

If there be any remainders, multiply the last remainder by the first divisor, and add the first remainder to the product; then write the whole original divisor below this, and the fraction thus formed is annexed to the quotient.

Example.—Divide 37255 by 63.

$$
\begin{array}{l}
9\overline{)37255} \\
\quad 7\overline{)4139\frac{4}{9}} \\
\qquad 591\frac{22}{63} \text{ Quot.}
\end{array}
$$

Here the *divisor*, 63, may be resolved into the two factors, 9 and 7; therefore divide first by 9, and then the quotient, 4139, by 7: the quotient, 591, is the answer required. Again, since the second remainder is 2, multiply it by 9, the first divisor, and to the product, 18, add 4, the first remainder; and the sum, 22, is the true remainder.

Exercises.

Divide the following numbers:—

1. 38297546	by 14	15	16		3. 90127539	by 22	24	25
2. 12345678	" 18	20	21		4. 16254973	" 32	36	42

III. WHEN THE DIVISOR IS A NUMBER HAVING *nothings* ANNEXED TO IT.

RULE.—Point off the nothings from the divisor, and also point off an equal number of figures from the *right* of the dividend; then divide by the remaining figures of the divisor. If these do not exceed 12, the question is wrought by *short* division. If above 12, *long* division is employed. To any remainders after the division is completed, annex the figures pointed off from the dividend, and mark the whole in the quotient, with the divisor written below it, to form a fraction.

Example.—Divide 73450 by 700.

$$
\begin{array}{l}
7,00\overline{)734,50} \\
\qquad 104\frac{650}{700} \text{ Ans.}
\end{array}
$$

Here the account being stated for short division, we point off the two nothings from the divisor, and 2 figures from the dividend. We then divide by 7; the answer is 104, and 6 over. Annex to this remainder the two figures which were pointed off from the dividend, 50, and the whole remainder is 650, below which the divisor is written, forming the fraction $\frac{650}{700}$.

Answers.

1. 2735539	$2553169\frac{11}{15}$	$2393596\frac{10}{16}$
2. 685871	$617283\frac{18}{20}$	$587889\frac{9}{21}$
3. $4096706\frac{7}{22}$	$3755314\frac{3}{24}$	$3605101\frac{14}{25}$
4. $507967\frac{29}{32}$	$451527\frac{1}{36}$	$387023\frac{7}{42}$

Exercises.

1. Divide	83473	by	900,	. . . . Answer	$92\frac{673}{900}$
2. 〃	〃	〃	260,	. . . . 〃	$321\frac{13}{260}$
3. 〃	8506126	〃	7400,	. . . . 〃	$1149\frac{3526}{7400}$
4. 〃	9013735	〃	8000,	. . . . 〃	$1126\frac{5735}{8000}$
5. 〃	1273068	〃	9600,	. . . . 〃	$132\frac{5868}{9600}$
6. 〃	3568405	〃	8100,	. . . . 〃	$440\frac{4405}{8100}$

NOTE.—WHEN THE DIVISOR IS 10, 100, 1000, or 1 with any other number of nothings annexed, the division is accomplished merely by pointing off as many figures from the right of the dividend as there are nothings in the divisor: the remaining figures are the quotient, and the figures pointed off, with the divisor written below them, form a fraction to be annexed to the quotient; for example—

To divide 1748 by 10, point off 1 figure, and the quotient is $174\frac{8}{10}$
〃 2376 〃 100, 〃 2 figures, 〃 〃 $23\frac{76}{100}$

IV. WHEN THE DIVISOR CONTAINS A FRACTION—AS $7\frac{3}{4}$.

A number with a fraction annexed, such as $7\frac{3}{4}$, is called a *mixed number*. Before we can divide by such a number, it must be converted into a simple one; that is, into a number having one denomination.

RULE.—Multiply the *whole* number, or integer of the divisor, by the lower figure of the fraction, and annex the upper figure to the product: multiply also the dividend by the lower figure of the fraction; then divide the one sum by the other.

Example.—Divide 5736 by $6\frac{3}{7}$.

$$
\begin{array}{l}
6\frac{3}{7})\ 5736 \\
\underline{7\qquad\ \ 7} \\
45)40152(892 \\
\quad\ \underline{360} \\
\quad\ \ \ 415 \\
\quad\ \ \ \underline{405} \\
\quad\ \ \ \ \ 102 \\
\quad\ \ \ \ \ \ \underline{90} \\
\quad\ \ \ \ \ \ 12 = \frac{12}{45}
\end{array}
$$

Here the divisor, 6, is multiplied by 7, the lower figure of the fraction, and 3, the upper figure, is added to the product, making 45; that is, $6\frac{3}{7}$ are the same as 45 *sevenths*.

In order, however, to preserve the proportion between the numbers, the sum to be divided must also be multiplied by 7, reducing it in like manner to sevenths. The two numbers being now both reduced to sevenths, the division takes place: the answer is 892, and the remainder is 12, which is expressed as $\frac{12}{45}$.

Exercises.

1. Divide	417638	by	$4\frac{2}{3}$,	. . . . . Answer	$89493\frac{12}{14}$
2. 〃	386543	〃	$8\frac{4}{5}$,	. . . . . . 〃	$43925\frac{15}{44}$
3. 〃	7695478	〃	$12\frac{5}{9}$,	. . . . . 〃	$612914\frac{20}{113}$
4. 〃	673685	〃	$14\frac{3}{4}$,	. . . . . . 〃	$45673\frac{33}{59}$
5. 〃	7103647	〃	$9\frac{5}{8}$,	. . . . . 〃	$738041\frac{19}{77}$
6. 〃	8301765	〃	$57\frac{3}{7}$,	. . . . . . 〃	$144558\frac{39}{402}$

COMPOUND NUMBERS.

Compound Numbers are those which consist of two or more kinds: when we speak of a *pound*, a simple number is expressed, it is one kind of thing; but when we speak of a sum consisting of *pounds*, *shillings* and *pence*, then we refer to various denominations or kinds; in other words, a sum *compounded*, of various kinds of money, and hence termed a compound number.

All questions which refer to sums of money, and to calculations of weights, measures, &c., which consist of various kinds or denominations, are placed under the head of *Compound* Numbers.

Calculations in *simple* numbers are the same in every country, and must continue without change throughout all time; for example, that 2 and 2 are equal to 4, is a universal truth, which all mankind cannot alter. The rules of Simple Addition, Subtraction, Multiplication, and Division, are therefore in use in every country in the world. With calculations in *compound* numbers, the case is entirely different, as almost every country has its own standards of money, weights, and measures, and the arithmetical rules for working differ accordingly. These calculations, however, could be rendered as easy as those in simple numbers, if the standards of money, weights, and measures, were constructed on the same decimal principle of advancing by tens.

In Great Britain and Ireland, the standards of money, weights, and measures are mixed and various, as may be learned from the following tables.

It is necessary that the pupil commit these tables to memory, before proceeding with the calculations in compound numbers.

ARITHMETICAL TABLES.

STERLING MONEY.

			Marks
		1 farthing.	¼
2 Farthings	=	1 half-penny.	½
4 Farthings	=	1 penny.	*d.*
12 Pence	=	1 shilling.	*s.*
20 Shillings, or 240 pence, or 960 farthings	=	1 pound.	£

A groat is 4 pence: a half-crown, 2 shillings and sixpence: a crown, 5 shillings: a half-sovereign, 10 shillings: a sovereign or pound, 20 shillings. The *guinea*, a coin now disused, but still often spoken of, consisted of 21 shillings.

All accounts are kept and reckoned by *pounds*, *shillings*, *pence*, and *farthings*. L. or £ is the initial letter of the Latin word *libra*, a pound, and is used to denote pounds; *s.*, from the Latin word *solidus*, stands for shillings; and *d.*, from *denarius*, for pence: £ *s. d.* are therefore respectively placed over columns of pounds, shillings, and pence. The mark for a half-penny is ½; for a farthing, ¼; and for three farthings, ¾.

The old Scottish pound was equal to 1s. 8d. sterling: hence £100 Scots was = £8, 6*s.* 8*d.* of our present money.

Note.—For an account of a new system of reckoning money decimally, proposed to be substituted for the present mode of reckoning; see Appendix.

AVOIRDUPOIS WEIGHT.

				Marks.
			1 dram.	*dr.*
16 Drams		=	1 ounce.	*oz.* = $437\frac{1}{2}$ grs. troy.
16 Ounces		=	1 pound.	*lb.* = 7000 " "
28 Pounds		=	1 quarter.	*qr.*
4 Quarters,	or 112 lbs.	=	1 hundredweight.	*cwt.*
20 Hundredwts.,	or 2240 lbs.	=	1 ton.	*T.*

A stone is 14 pounds: 8 stones, a hundredweight.

All ordinary articles are weighed by this weight, which is known as the imperial standard, and is in universal use in this country.

TROY WEIGHT.

				Marks.
			1 grain.	*gr.*
24 Grains		=	1 pennyweight.	*dwt.*
20 Dwts.,	or 480 grains	=	1 ounce.	*oz.*
12 Ounces,	or 5760 "	=	1 pound.	*lb.*

Gold, silver, and precious stones, are weighed by troy weight. In determining the purity of gold, the gold is supposed to be divided into 24 carats, and if pure, is said to be 24 carats fine; if there be 23 carats of pure gold, and 1 of alloy, it is said to be 23 carats fine; and so on.

APOTHECARIES' WEIGHT.

			Marks.
		1 grain.	*gr.*
20 Grains	=	1 scruple.	*sc.* or ℈
3 Scruples	=	1 dram.	*dr.* " ʒ
8 Drams	=	1 ounce.	*oz.* " ℥
12 Ounces	=	1 pound.	*lb.*

The ounce and pound are the same as in the troy weight. This weight is used only in preparing medicines.

LINEAL MEASURE, OR MEASURE OF LENGTH.

				Marks.
			1 inch.	*in.*
12 Inches		=	1 foot.	*ft.*
3 Feet,	or 36 inches	=	1 yard.	*yd.*
$5\frac{1}{2}$ Yards		=	1 perch or pole.	*per.*
40 Poles,	or 220 yards	=	1 furlong.	*fur.*
8 Furlongs,	or 1760 yards, or 5280 feet	=	1 mile.	
3 Miles		=	1 league.	

A fathom is 6 feet: a hand, 4 inches.

For smaller lengths than an inch, eighths and sixteenths of an inch, also tenths, are used.

This is the ordinary measure for dimensions of all kinds. Distances are measured by miles: depth by fathoms: the height of horses by hands. The chain used for measuring land = 66 feet, and consists of 100 links, each of them 7·92 inches.

CLOTH MEASURE.

			Marks.
$2\frac{1}{4}$ Inches	. . . . .	= 1 nail.	*nl.*
4 Nails,	or 9 inches .	= 1 quarter.	*qr.*
4 Quarters,	or 36 " . .	= 1 yard.	*yd.*
5 Quarters,	or 45 " .	= 1 English ell.	

The yard in this measure is of the same length as that in Lineal Measure.

YARN MEASURE.

COTTON.		LINEN.	
54 Inches . .	= 1 thread.	90 Inches .	= 1 thread.
80 Threads .	= 1 skein.	120 Threads .	= 1 cut.
7 Skeins . .	= 1 hank.	2 Cuts . .	= 1 hear.
18 Hanks .	= 1 spindle.	6 Hears .	= 1 hasp.
		4 hasps . .	= 1 spindle.

SQUARE OR SUPERFICIAL MEASURE.

144	Square inches	= 1 square foot.
9	Square feet	= 1 " yard.
$30\frac{1}{4}$	Square yards	= 1 " perch.
40	Square perches	= 1 rood.
4	Roods	= 1 acre.
640	Acres	= 1 square mile.

36 square yards are 1 rood of building: 100 square feet, 1 square of flooring: $272\frac{1}{4}$ square feet, 1 rood of Bricklayer's work: 10,000 square links, 1 square chain: 10 square chains, 1 acre.

Superficial measure refers to breadth as well as length. For instance, a *superficial foot*, usually called a square foot, is either a square of a foot in length and a foot in breadth—in other words, a foot each way—or it is any dimension in which the length multiplied by the breadth will form a foot, or 12 inches, each way: thus, 12 times 12 inches = 144 inches, is a square foot; or 6 times 24 inches = 144, is also a square foot. Sometimes the term *square feet* is confounded with that of *feet square*, which is quite a different thing. A piece of cloth said to measure six square feet, consists of six squares of a foot each; but a piece said to measure *six feet square* would be six feet along each side, and comprise thirty-six squares of a foot each. Inattention to these distinctions has often led to awkward errors and disputes.

It will now be understood, that the *square* of any number is that number multiplied by itself; thus—the square of 3 is 3 times 3, or nine; the square of 4 is 16; and so on. On this principle is formed the table of square measure, for the measurement of breadth and length: it is commonly used for measuring land, walls, &c.

CUBIC OR SOLID MEASURE.

1728	Cubic inches	= 1 cubic foot.
27	" feet, or 46,656 inches	= 1 " yard.
40	" "	= 1 ton shipping.
5	" "	= 1 barrel bulk.

40 Cubic feet of rough timber, or 50 of hewn timber, is a load.

Solid measure is computed by multiplying the length by the breadth, and the product by the thickness. This measure is used in calculating the solid contents of masses of earth, &c.; in measuring the holds of vessels, to ascertain the tonnage; and in all cases where length, breadth, and thickness are reckoned.

LIQUID MEASURE OF CAPACITY.

			Marks.
4 Gills	=	1 pint.	*pt.*
2 Pints	=	1 quart.	*qt.*
4 Quarts, or 8 pints, or 32 gills	=	1 gallon.	*gal.*

A hogshead (hhd.) contains 63 gallons. A pipe is 2 hogsheads, and 2 pipes form a tun. But in trade these measurements are not rigidly adhered to, as casks differ in capacity. 1 gill = 5 oz. avoirdupois of water, or about 8¾ cubic inches. All liquids are measured by this table.

GRAIN MEASURE, OR DRY MEASURE OF CAPACITY.

			Marks.
2 Gallons	=	1 peck.	*pk.*
4 Pecks, or 8 gallons	=	1 bushel.	*bu.*
8 Bushels	=	1 quarter.	*qr.*

5 bushels are a sack: 5 quarters, a load.

The gallon is the same as in Liquid Measure. By this table, grain, seeds, flour, &c., are measured.

TABLE OF TIME.

60 Seconds	=	1 minute.
60 Minutes	=	1 hour.
24 Hours	=	1 day.
7 Days	=	1 week.
52 Weeks 1 day, or 365 days	=	1 ordinary year.
365 Days, 5 hours, 48 minutes, 49 seconds	=	1 solar year.
366 Days	=	1 leap year.

The year is also divided into 12 Calendar Months; namely—

January,	31 days.	May,	31 days.	September,	30 days.
February,	28 "	June,	30 "	October,	31 "
March,	31 "	July,	31 "	November,	30 "
April,	30 "	August,	31 "	December,	31 "

The number of days in each month may be easily remembered from the following well-known lines:—

Thirty days have September,
April, June, and November;
All the rest have thirty-one,
Excepting February alone,
Which hath but twenty-eight days clear,
And twenty-nine in each leap-year.

As the true solar year is nearly 6 hours more than 365 days, every fourth year, termed leap-year, is reckoned as consisting of 366 days, in order to make allowance for the excess; the additional day being given to February.

To ascertain if a given year is leap-year, divide it by 4, and if there is no remainder, it is leap-year; if there be a remainder, the number over indicates how many years it is *after* leap-year. Thus, 1852 is leap-year, because divisible by 4 without a remainder; and 1854 is two years after leap-year, because there is a remainder of 2, after dividing it.

GEOGRAPHICAL OR NAUTICAL MEASURE.

1 Geographical mile . .	=	$1\frac{3}{20}$ imp. mile, or 6076 feet.
3 〃 miles .	=	1 league.
60 〃 miles . .	=	1 degree. marked *deg.* or [°]
360 〃 degrees, or about 24,855½ imperial miles	=	The circumference of the earth.

The degree is divided into 60 minutes (marked ′), and the minute into 60 seconds (marked ″). This measure is used for geographical purposes, and in reckoning distances at sea. At the equator, a degree of longitude is 69⅙ imperial miles.

MISCELLANEOUS.

Aum of hock wine .	=	30 gals.	Hogshead of rum .	=	45 to 50 gals.
Bag of coffee .	=	1¼ to 1½ cwt.	〃 sugar	=	13 to 16 cwt.
〃 hops . .	=	about 2½ 〃	Keel of coal . .	=	21 tons.
〃 rice (E. I.)	=	about 1½ 〃	Load of bricks .	=	500 bricks.
Bale of coffee (Mocha)	=	2 to 2½ 〃	〃 tiles . .	=	1000 tiles.
〃 cotton . .	=	200 to 500 lbs.	Loaf (quartern) .	=	4 lbs.
〃 flax (Russia)	=	5 to 6 cwt.	Mat of flax (Dutch)	=	126 〃
Barrel of beef .	=	200 lbs.	Pipe of Port wine .	=	115 gals.
〃 beer .	=	36 gals.	〃 Madeira .	=	92 〃
〃 butter .	=	224 lbs.	Pocket of hops . .	=	1½ to 2 cwt.
〃 flour .	=	196 〃	Puncheon of brandy	=	100 to 110 gals.
〃 gunpowder	=	100 〃	〃 rum .	=	90 to 100 〃
〃 herring .	=	500 her.	〃 whisky	=	about 120 〃
〃 soap (soft)	=	256 lbs.	Quintal of fish .	=	112 lbs.
〃 tar . .	=	26¼ gals.	Quire of paper *	=	24 sheets.
Box of raisins .	=	56 lbs.	Ream of paper .	=	20 quires, or 480 sheets.
〃 salmon . .	=	120 to 130 〃			
Butt of sherry .	=	108 gals.	Robin of coffee .	=	1 to 1½ cwt.
Case of mace . .	=	about 1½ cwt.	Roll of parchment	=	60 skins.
Cask of clover .	=	7 to 9 〃	Sack of clover .	=	2 to 3½ cwt.
〃 raisins .	=	1 to 2½ 〃	〃 flour .	=	280 lbs.
〃 rice (Amer.)	=	6 〃	Score . . .	=	20 articles.
Chaldron of coal .	=	25½ 〃	Seron of almonds	=	1¼ to 2 cwt.
Chest of soap .	=	3¼ 〃	Sheet of paper folded—		
〃 tea, congou	=	about 84 lbs.	into 2 leaves,	is termed	folio size.
〃 〃 hyson	=	60 to 80 〃	〃 4 〃	is termed	4to, or quarto.
Dozen . . .	=	12 articles.	〃 8 〃	is termed	8vo, or octavo.
Drum of figs .	=	24 lbs.	〃 12 〃	is termed	12mo, or duodecimo.
Firkin of beef . .	=	100 〃	〃 16 〃	is termed	16mo.
〃 butter .	=	56 〃	〃 18 〃	is termed	18mo.
Fodder of lead . .	=	19½ cwt.	〃 24 〃	is termed	24mo.
Frail of figs . .	=	32 to 56 lbs.	〃 48 〃	is termed	48mo.
Gross . . .	=	12 doz.	Tierce of coffee .	=	5 to 7 cwt.
Hogshead of beer .	=	54 gals.	〃 sugar .	=	7 to 9 〃
〃 brandy	=	60 〃	Tub of butter . .	=	84 lbs.

* In Scotland, a quire consists of 24 sheets of folio paper, but of 48 sheets of quarto and octavo size.

TERMS FOR LEASES, ETC.

ENGLAND AND IRELAND.			SCOTLAND.		
Lady-day . .	March	25	Candlemas . .	February	2
Midsummer .	June	24	Whitsunday .	May	15
Michaelmas . .	September	29	Lammas . .	August	1
Christmas .	December	25	Martinmas .	November	11

When a Scottish Term falls on Sunday, the Monday following is considered Term-day.

REDUCTION.

Reduction is the method of converting sums in Compound Numbers from a higher to a lower denomination, or from a lower to a higher; as, for instance, from pence to pounds, or from pounds to pence. The calculations are made by means of the preceding Arithmetical Tables, which shew how many pence there are in a certain number of pounds, or pounds in a certain number of pence; the number of feet in so many yards, or yards in so many feet; and so on.

I. To convert numbers from a higher to a lower Denomination.

Rule.—1. Multiply the highest denomination in the given sum, by the number of times that the next lower denomination is contained in *one* of the higher, adding to the product any of the lower denomination in the given sum: thus—to convert £3, 15*s.* 0*d.* into shillings, multiply the 3 pounds by 20, the number of shillings in a pound, and add the 15*s.* to the product, making in all 75*s.*

2. If the reduction is to be carried further, convert this product into the next lower denomination, by multiplying, &c., in a similar way: thus—to convert 75*s.* 3*d.* into pence, multiply the 75*s.* by 12, the number of pence in a shilling, adding the 3*d.* to the product, making 903 pence; and so on with each denomination, till the whole has been reduced to the lowest given term.

Example 1.—Reduce £4 to shillings.

£4	
20	
80 *shillings.*	
12	
960 *pence.*	

Here there being 20 shillings in *one* pound, we multiply by 20; the answer is 80 shillings, which is the number contained in £4. If it should be required further to reduce the shillings to pence, we multiply 80 by 12, as there are 12 pence in one shilling; the result is 960, which is the number of pence in 80 shillings, or £4.

Example 2.—Reduce £3, 14*s.* 7¼*d.* to farthings.

£	s.	d.
3	14	7¼
20		
74 *shillings.*		
12		
895 *pence.*		
4		
3581 *farthings.*		

Here the pounds are brought first to shillings, then the shillings to pence, and the pence to farthings; and as, besides the £3, there are 14*s.* 7¼*d.*, these are to be added at the successive reductions: in multiplying the pounds by 20, we include the 14*s.*, making 74 shillings; multiplying these again by 12, we include the 7*d.*, making 895 pence, and these are reduced to farthings by multiplying by 4, and including the ¼, making 3581 farthings.

Note.—To convert guineas into pounds, multiply the number of guineas by 21, as there are 21 shillings in a guinea, and divide the product by 20, the number of shillings in a pound.

II. To convert numbers from a lower to a higher Denomination.

Rule.—1. Divide the given denomination by the number of times that it is contained in *one* of the next higher denomination, the sum is thus converted to the denomination immediately above it: thus, to convert 345 farthings into pence, divide by 4, the number of farthings in a penny, and the answer is 86 pence, and 1 farthing over.

2. If the conversion is to be carried further, divide the new denomination by the number of times that it is contained in *one* of the denomination above it: thus, to convert 86 pence into shillings, divide by 12, the number of pence in a shilling, and the answer is 7*s.* and 2 pence over; and so on, dividing each denomination in succession, till the sum has been converted to its highest required denomination.

The remainders at each stage of the division are of the same denomination as the number from which they arise, and must be marked as such in the answer; thus, in the above example, the answer is 7 shillings, 2 pence, 1 farthing, or, 7*s.* 2¼*d.*

Example 1.—In 3840 farthings, how many pounds?

4)3840 *farthings.*
12)960 *pence.*
20)80 *shillings.*
4 *pounds.*

The farthings are first converted into pence, by dividing the number by 4, because there are four farthings in one penny: to convert pence into shillings, divide by 12, since twelve pence are one shilling: and to convert shillings into pounds, divide by 20, as twenty shillings are one pound.

Example 2.—Reduce 5863 farthings to pounds.

4)5863 *farthings.*
12)1465, ¾
20)122, 1
£6, 2*s.* 1¾*d.*

The number of farthings, or pence, or shillings, to be brought to a higher denomination, is not always an exact number of that denomination; there may be a remainder after each division, and here, in bringing the farthings to pence, there is a remainder of 3—that is, of 3 farthings; again, in bringing the pence to shillings, 1 remains—that is, 1 penny: and in bringing the shillings to pounds, there is a remainder of 2 shillings; we find thus that 5863 farthings make £6, 2*s.* 1¾*d.*

All questions of reduction of money, weights, measures, &c., whether from a higher to a lower, or from a lower to a higher denomination, are wrought by these two rules of Reduction.

Exercises in Reduction of Money.

1. Reduce £15 to pence, ...	Ans.	3600 pence.
2. " £73, 14*s.* 8*d.* to pence, .	"	17696 pence.
3. In £374, 15*s.*, how many shillings? .	"	7495 shillings.
4. In £253, 9*s.* 6¼*d.*, how many farthings?	"	243338 farthings.
5. Reduce £7, 19*s.* 5½*d.* to halfpence, .	"	3827 halfpence.

Exercises in Reduction of Money.

6.	Reduce £73, 12*s.* 6*d.* to sixpences,	Ans.	2945 sixpences.
7.	In 257 crowns, how many shillings?	"	1285 shillings.
8.	In 324 guineas, how many pence?	"	81648 pence.
9.	In 836 pence, how many shillings?	"	69*s.* 8*d.*
10.	In 9568 farthings, how many pounds?	"	£9, 19*s.* 4*d.*
11.	Reduce 875 shillings to crowns, .	"	175 crowns.
12.	In 83765 farthings, how many guineas?	"	83 guineas, 2*s.* 1¼*d.*
13.	Reduce to farthings, £17, 13*s.* 4½*d.*	"	16962 farthings.
14.	" " £63, 13*s.* 11¾*d.*	"	61151 "
15.	" " £302, 5*s.* 9¼*d.*	"	290197 "
16.	" halfpence, £34, 14*s.* 7*d.*	"	16670 halfpence.
17.	" " £207, 4*s.* 3½*d.*	"	99463 "
18.	" " £7624, 13*s.* 8½*d.*	"	3659849 "
19.	Convert to pounds, 7314 farthings,	"	£7, 12*s.* 4½*d.*
20.	" " 59633 "	"	£62, 2*s.* 4¼*d.*
21.	" " 71058 "	"	£74, 0*s.* 4½*d.*
22.	" " 9468 guineas,	"	£9941, 8*s.* 0*d.*

Avoirdupois Weight.

Example 1.—Reduce 15 tons to ounces.

$$\begin{array}{rl} 15 & \textit{tons.} \\ 20 & \\ \hline 300 & \textit{cwts.} \\ 4 & \\ \hline 1200 & \textit{qrs.} \\ 28 & \\ \hline 9600 & \\ 2400 & \\ \hline 33600 & \textit{lbs.} \\ 16 & \\ \hline 201600 & \\ 33600 & \\ \hline \textit{Ans. } 537600 & \textit{oz.} \end{array}$$

This is reduction from a higher to a lower denomination, and is done by *multiplying*. The tons are first to be reduced to hundredweights, and are multiplied by 20, because there are 20 cwts. in one ton; the cwts. are then brought to quarters by multiplying by 4; the quarters to pounds by multiplying by 28; and, lastly, the pounds to ounces, by multiplying by 16. We find, therefore, that in 15 tons there are 300 cwts., or 1200 qrs., or 33600 lbs., or 537600 ounces.

Example 2.—Reduce 7 tons 8 cwt. 2 qrs. 15 lbs. to pounds.

$$\begin{array}{rlll} \textit{Tons.} & \textit{cwts.} & \textit{qrs.} & \textit{lbs.} \\ 7 & 8 & 2 & 15 \\ 20 & & & \\ \hline 148 & \textit{cwts.} & & \\ 4 & & & \\ \hline 594 & \textit{qrs.} & & \\ 28 & & & \\ \hline 4767 & & & \\ 1188 & & & \\ \hline \textit{Ans. } 16647 & \textit{lbs.} & & \end{array}$$

In this example, in bringing the tons to cwts., the 8 cwts. in the sum are included, making 148 cwts.; again, in reducing the cwts. to quarters, the 2 qrs. are included, which give 594 qrs.; and, lastly, in bringing the quarters to pounds, the 15 lbs. are included, making in all 16647 pounds.

Example 3.—Convert 54659 lbs. to tons.

```
lbs.      4
28)54659(1952. 3
   28    20)488. 0
   266   tons 24. 8. 0. 3
   252
    145
    140
     59
     56
      3
```

[The 4 is put over the 1952, as this is the most convenient situation in which to place it.]

This is reduction from a lower to a higher denomination, and is done, by *dividing*. The pounds are divided by 28, to bring them to quarters, since there are 28 lbs. in one quarter; the answer is 1952 quarters, and 3 lbs. over. The quarters are now divided by 4, to bring them to cwts.; the answer is 488 cwts., and no remainder. The cwts. are now divided by 20, to make tons, and the answer is 24 tons, and 8 cwts. over. We find, thus, that in 54659 lbs. there are 24 tons 8 cwts. 0 qrs. 3 lbs.

Exercises in Avoirdupois Weight.

1. In 8 tons 7 cwt., how many lbs.? . . Ans. 18704 lbs.
2. In 3 cwts. 2 qrs. 13 lbs., how many ounces? 〃 6480 oz.
3. Reduce 7865 lbs. to tons. Ans. 3 tons 10 cwts. 0 qrs. 25 lbs.
4. 〃 87643 ounces to cwts. 〃 48 cwts. 3 qrs. 17 lbs. 11 oz.
5. 〃 8764 drams to lbs. 〃 34 lbs. 3 oz. 12 drams.
6. 〃 735 tons 12 cwts. 3 qrs. to lbs. Ans. 1647828 lbs.
7. Bought 15 chests of tea, which weighed each 13 cwts. 2 qrs. 17 lbs. How many lbs. were there in the 15 chests? Ans. 22935 lbs.
8. In a hogshead of sugar, weighing 13 cwts. 2 qrs. 17 lbs., how many lbs.? . . . . Ans. 1529 lbs.
9. How many lbs. are there in 8375 ounces? Ans. 523 lbs. 7 oz.
10. The quantity of tea used in Great Britain is reckoned at about 36 millions of lbs. every year; how many tons is that? Ans. 16071 tons 8 cwts. 2 qrs. 8 lbs.
11. Convert 17 tons 13 cwts. of sugar to lbs. . Ans. 39536 lbs.
12. 〃 76123 ounces to cwts., Ans. 42 cwts. 1 qr. 25 lbs. 11 oz.

Troy Weight.

Example 1.—Reduce 3 lbs. 5 oz. 14 dwts. 16 grs. to grains.

```
  lbs. oz. dwts. grs.
   3    5   14   16
  12
  41 oz.
   20
  834 dwts.
   24
  3352
 1668
Ans. 20032 grains.
```

This is reducing from a higher to a lower denomination. Therefore, *multiplying* the lbs. by 12, because there are 12 ounces in the lb., and then adding 5, we have 41 oz. These are multiplied by 20, to bring to dwts., and 14 are added, giving 834 dwts., which, being multiplied by 24, and 16 added, give 20032 grains.

Example 2.—Reduce 20032 grains to lbs.

```
     Grains  20
24)20032(834. 16
   192    12)41. 14
   ----
    83   lbs. 3. 5. 14. 16
    72
   ----
   112
    96
   ----
    16
```

This is just the reverse operation of the former, and is done by division. The grains are divided by 24, to bring them to dwts., and the result is 834 dwts. and 16 grains. The dwts. divided by 20, gives 41 oz. 14 dwts. Finally, the oz. divided by 12, gives 3 lbs. 5 oz.; to this annex the other remainders, and the answer is 3 lbs. 5 oz. 14 dwts. 16 grs.

Exercises in Troy Weight.

1. In 4 lbs. 7 oz. of gold, how many ounces? . . Ans. 55 oz.
2. In 3765 grains of gold-dust, how many ounces?
   Ans. 7 oz. 16 dwts. 21 grs.
3. In 7 lbs., how many ounces, pennyweights, and grains?
   Ans. 84 oz., or 1680 dwts., or 40320 grs.
4. Reduce 84627 grains to lbs. Ans. 14 lbs. 8 oz. 6 dwts. 3 grs.
5. 〃 8 oz. 13 dwts. to grains, . . Ans. 4152 grains.
6. 1 dozen silver table-spoons weighs 976 pennyweights, how many pounds do they weigh? . Ans. 4 lbs. 0 oz. 16 dwts.

Lineal Measure.

1. Reduce 5 miles 7 furlongs to perches, . Ans. 1880 perches.
2. 〃 76 yards 2 feet 7 inches to inches, 〃 2767 inches.
3. 〃 76854 inches to yards, . Ans. 2134 yds. 2 ft. 6 in.
4. In 847365 yards, how many miles? 〃 481 miles 805 yds.
5. In 75061 feet, how many miles? . 〃 14 miles 380 yds. 1 ft.
6. In 87 miles 3 furlongs 15 perches 2 yards, how many yards?
   Ans. 153864½ yards.

Cloth Measure.

1. Reduce 836 yards to nails, . . . Ans. 13376 nails.
2. 〃 13376 nails to yards, . . . 〃 836 yds.
3. In 56 yards 2 qrs. 3 nails, how many nails? 〃 907 nails.
4. In 7463 nails, how many yards? Ans. 466 yds. 1 qr. 3 nails.
5. In 367 English ells, how many yards? Ans. 458 yds. 3 qrs.
6. In 534 yards, how many ells English? 〃 427 ells 1 qr.

Square Measure.

1. Reduce 276 acres 3 ro. 17 per. to yards, Ans. 1339984¼ sqr. yds.
2. 〃 87654 perches to acres, . 〃 547 ac. 3 ro. 14 per.
3. 〃 4785694 square inches to square yards,
   Ans. 3692 yds. 5 feet 142 inches.
4. In a field containing 67654 square yards, how many acres?
   Ans. 13 acres 3 roods 36 per. 15 yds.
5. In a room 25 feet long by 18 feet broad, how many square yards? . . . . . Ans. 50 square yards.

Cubic Measure.

1. In 1259712 cubic inches, how many cubic yards? Ans. 27 yds.
2. In 235 cubic yards, how many cubic inches? Ans. 10905160 inches.
3. In 2280 cubic feet, how many tons of shipping? Ans. 57 tons.
4. How many solid yards in 391256 solid inches? Ans. 8 yds. 10 ft. 728 in.

Liquid Measure of Capacity.

1. Reduce 875 gallons to pints, . . . Ans. 7000 pints.
2. " 187564 gills to gallons, Ans. 5861 galls. 1 quart 1 pint.
3. In 76854 pints, how many gallons? Ans. 9606 galls. 3 quarts.
4. In 6 hogsheads, how many quarts? . . . Ans. 1512 quarts.

Grain Measure, or Dry Measure of Capacity.

1. Reduce 56 quarters 5 bushels to pecks, . Ans. 1812 pecks.
2. " 7684 pecks to quarters, . . Ans. 240 qrs. 1 bush.
3. In 79 quarters 3 bushels 2 pecks 1 gall., how many gallons? Ans. 5085 gallons.
4. In 648 quarters of wheat, how many pecks? " 20736 pecks.

Time.

1. Reduce 26 days 7 hours to minutes, . Ans. 37860 min.
2. " 5 days 4 hours 15 minutes to seconds, " 447300 sec.
3. In 876543 days, how many years? Ans. 2401 years 178 days.
4. In 17 weeks 3 days 7 hours, how many hours? Ans. 2935 ho.
5. How many hours has a boy lived who is 9 years and 15 weeks old? . . . . Ans. 81144 hours.

Miscellaneous.

1. How many pounds in 78543 fourpences? . Ans. £1309, 1s.
2. How many lbs. in 26 tons 3 cwts. 1 qr. of soap? " 58604 lbs.
3. How many lbs. in 35726 dwts. of silver? Ans. 148 lbs. 10 oz. 6 dwts.
4. In 23 miles 2 fur. 219 yds., how many feet? Ans. 123417 ft.
5. How many yards in 78325 nails of cloth? Ans. 4895 yds. 1 qr. 1 nl.
6. In 73926 inches, how many miles? Ans. 1 m. 1 fur. 73 yds. 1 ft. 6 in.
7. How many poles in 76 acres 1 rood 39 poles of land? Ans. 12239 poles.
8. In 56 gallons 3 pints of water, how many gills? Ans. 1804 gs.
9. How many pecks in 208 quarters 3 bushels of wheat? Ans. 6668 pecks.
10. If a steamer sail 240 nautical miles in 24 hours, how many imperial miles is that per hour? . Ans. $11\frac{1}{2}$ imp. miles.

COMPOUND ADDITION.

COMPOUND ADDITION is the adding together of Compound Numbers.

RULE.—1. Write down the given numbers in such a way that those of the same kind or denomination may stand directly below one another—pounds under pounds, shillings under shillings, &c.

2. Add the numbers in the lowest denomination, and find how many of the next higher denomination are contained in the amount. Carry the quotient to the next column, marking any remainder below the column just added.

3. Add the numbers of the next denomination in the same way, including what has been brought from the previous column, and so on, till all the denominations have been added. The highest denomination is added as in Simple Addition.

Example 1.—Add together the following sums—£31, 12*s.* 7½*d.*, £73, 14*s.* 8½*d.*, £69, 17*s.* 5¾*d.*, £87, 15*s.* 6¼*d.*, and £37, 12*s.* 3½*d.*

£	*s.*	*d.*
31	12	7½
73	14	8½
69	17	5¾
87	15	6¼
37	12	3½
£300	12	7½

Here, after writing down £ *s.* *d.*, place the pounds under £, the shillings under *s.*, and the pence with the farthings under *d.*; and draw a line under the figures. Beginning with the farthings, we have first ½, or 2 farthings, then 1, 3, 2, 2—in all, 10 farthings. These are converted to pence, and make 2½*d.*; put down the ½, and carry 2*d.* to the pence column. This being added, makes 31 pence, or 2 shillings and 7 pence: write the 7 under the pence, and carry the 2 to the unit row of shillings; adding it, we have 2, 2, 5, 7, 4, and 2—in all, 22: then, going to the next column, the figures in which are all *tens*, we say 22 and 10 are 32, then 42, 52, 62, 72—in all, 72 shillings, which, being converted to pounds, make £3, 12*s.*; 12, therefore, is written under the shillings, and 3 carried to the pounds, which are now added, and the amount put down as in Simple Addition.

NOTE.—In adding the *shilling* column, the following is the most convenient method:—Add the unit row of shillings as already described: in the foregoing example, it amounts to 22; mark the last figure, 2, below the unit row, and carrying the tens, 2, to the *tens'* row, add it also: then halve the amount (that is, divide it by 2); if 1 remains after halving, mark 1 below the tens' row, and carry the half to the pounds. If nothing remains on halving, carry the half to the pounds, but without marking anything below the tens' row of shillings. In the present example, the tens' row amounts to 7, which, being halved, makes 3, and 1 over; therefore mark the 1 below the tens' row of shillings, and carry 3 to the pounds.

The halving of the tens' row of shillings is merely a short way of dividing the amount of the shillings by 20, to convert them to pounds.

Example 2.—Add together 3 tons 7 cwts. 2 qrs. 17 lbs. 5 oz.; 7 tons 3 cwts. 2 qrs. 13 lbs.; 8 tons 12 cwts. 3 qrs. 4 lbs. 7 oz.; 6 tons 17 cwts. 2 qrs. 10 lbs. 9 oz.; 12 tons 9 cwts. 2 qrs. 18 lbs. 13 oz.

tons.	*cwts.*	*qrs.*	*lbs.*	*oz.*
3	7	2	17	5
7	3	2	13	0
8	12	3	4	7
6	17	2	10	9
12	9	2	18	13
38	11	1	8	2

Here having written down in a line the marks for tons, hundredweights, quarters, pounds, and ounces, write each quantity under its proper name, and draw a line under all. Commencing to add at the ounces, we find there are 34; now, 16 ounces being a pound, 34 oz. are equal to 2 lbs. and 2 oz.: put down the 2 oz. under the ounces, and carry the 2 lbs. to the pound column. Adding this column, there are 64 lbs., which, being converted to quarters, gives 2 qrs. and 8 lbs. over; the 8 lbs. are written under the pounds, and the 2 qrs. carried to the column of quarters. This being added, gives 13; and 13 qrs. converted to cwts., give 3 cwts. and 1 qr.; put down 1 under the qrs., and carry 3 to the cwts. Adding these, we find there are 51 cwts., which, being converted to tons, give 2 tons, and 11 cwts. over; write 11 under the cwts., and carry 2 to the tons. These are now added, and their amount put down as in Simple Addition.

All compound quantities being added on the same principles as the above, it is unnecessary to give examples of the addition of the other weights and measures.

Exercises in Money.

1.			2.			3.			4.		
£27	13	4½	£83	15	11½	£25	14	6	£36	15	7¼
39	6	5¼	24	15	9¼	97	13	11¾	98	14	10
28	17	9	61	18	2¾	25	18	10½	36	19	11½
64	1	11¾	97	14	7½	31	2	7½	42	13	8¾
27	19	10	27	6	10¼	69	13	4¾	70	14	5
91	13	9½	35	13	6½	48	16	2	49	17	3¼

5.			6.			7.			8.		
£47	16	8½	£73	14	9½	£123	7	3¾	£237	13	5
99	15	11¼	62	10	10	76	19	11½	315	18	4¼
47	10	5¾	50	3	8¼	9	10	10¼	78	17	5¾
53	14	9	75	13	7½	17	8	7½	250	10	10
81	15	6½	86	12	3¾	79	13	4¾	371	3	8½
59	18	3¾	79	17	6½	83	16	5½	93	13	7¼

Answers.

1. £274 13 2
2. 331 4 11¾
3. 298 19 6½
4. £335 15 9¾
5. 390 11 8¾
6. 428 12 9½
7. £390 16 7¼
8. 1347 17 4¾

9.			10.			11.			12.		
£373	15	10¼	£371	2	2½	£727	13	4¾	£4763	14	5
417	13	8	506	10	10¼	837	17	11½	3168	5	7½
350	4	11½	317	11	5	586	19	10¼	6732	5	9¼
78	15	5	75	7	11¾	398	15	9½	968	14	7½
314	8	6¾	961	14	5¼	613	12	11¾	7320	10	4¼
736	7	9	276	13	7¼	395	9	7½	8432	15	7¼
840	10	7½	354	6	8½	593	19	4¾	950	14	3½
514	3	8¼	763	5	2	675	12	11½	3765	4	7½

13.			14.			15.			16.		
£785	13	1	£697	12	6¾	£832	2	10½	£695	10	11½
392	10	10¾	385	17	9½	756	17	9¼	596	7	5
583	9	7½	583	19	5¾	647	11	3¼	372	19	10¾
694	5	5½	385	10	4½	392	13	11¾	847	8	3
257	3	9¾	716	17	11¼	238	19	4½	593	13	10½
395	17	11¼	167	14	7¼	375	15	11¼	392	5	9¼
287	12	6¾	973	3	10½	932	10	7¼	795	19	11¾
375	19	8½	286	19	9¾	657	6	3½	258	8	6½
987	8	5¾	682	2	5¼	746	9	10½	683	16	7¾

17.			18.			19.			20.		
£672	4	5¾	£634	14	9½	£789	19	11¾	£665	12	9½
267	13	9½	346	12	11¾	210	0	1¼	426	13	11
672	15	11¼	683	13	7¼	567	17	3¾	273	4	7
935	14	5½	267	7	8½	763	13	8½	129	3	8
593	18	10¾	695	19	9¼	649	17	6¼	791	17	5
359	11	1¼	999	7	5¾	783	12	8½	863	15	7½
111	11	11¼	842	16	10½	594	10	10½	473	12	5
369	14	7	795	5	5¾	279	5	3¾	926	19	7½
583	9	10½	689	17	8½	659	8	5	137	18	4

Avoirdupois Weight.

21.			22.			23.			24.		
tons.	*cwts.*	*qrs.*	*cwts.*	*qrs.*	*lbs.*	*lbs.*	*oz.*	*dr.*	*lbs.*	*oz.*	*drs.*
7	13	2	7	3	15	23	7	6	13	13	6
3	15	3	13	2	17	17	13	5	17	12	15
12	7	1	9	1	9	21	10	12	26	7	11
23	11	3	14	2	13	9	14	7	14	9	7
17	12	2	16	3	18	12	6	8	11	12	3

Answers.

9.	£3626	0	6¼		13.	£4760	1	6¾		17.	£4566	15	0¾
10.	3626	12	4½		14.	4879	18	10½		18.	5955	16	4¾
11.	4830	1	11½		15.	5580	7	11¾		19.	5298	5	11¼
12.	36102	5	3¾		16.	5236	11	4		20.	4688	18	4½

21. 65 tons 0 cwts. 3 qrs.
22. 62 cwts. 1 qr. 16 lbs.
23. 85 lbs. 4 oz. 6 drams.
24. 84 lbs. 7 oz. 10 drams.

Avoirdupois Weight.

25.			26.		
cwts.	*qrs.*	*lbs.*	*tons.*	*cwts.*	*qrs.*
437	3	11	68	19	1
743	1	13	34	3	3
695	2	26	72	15	2
217	2	19	97	11	1
416	3	27	9	17	0
932	2	5	35	7	3

Troy Weight.

27.				28.		
lbs.	*oz.*	*dwts.*	*grs.*	*lbs.*	*oz.*	*dwts.*
17	5	19	23	3	11	19
23	11	16	21	5	7	13
96	5	7	22	4	9	15
15	9	3	7	7	10	7
4	3	18	19	8	5	6
17	10	15	14	9	7	3

Lineal Measure.

29.			30.		
miles.	*fur.*	*per.*	*per.*	*yds.*	*ft.*
317	7	25	27	3	2
83	5	14	39	2	1
132	4	27	34	4	2
75	6	32	18	3	2
97	2	18	28	5	1
57	4	23	35	3	2

Cloth Measure.

31.			32.			
yds.	*qrs.*	*nails.*	*yds.*	*qrs.*	*nls.*	*in.*
35	3	2	95	3	2	1
47	2	3	73	2	3	2
73	1	2	39	1	2	2
59	3	1	46	2	1	1
64	2	3	84	3	2	2
51	1	2	69	2	3	1

Square and Land Measure.

33.			34.		
ac.	*ro.*	*per.*	*per.*	*yds.*	*ft.*
327	3	25	34	23	2
435	2	17	16	14	4
706	1	34	26	12	2
69	2	26	32	25	7
75	3	29	19	13	2
83	2	35	28	22	8

Liquid Measure.

35.			36.		
gals.	*qrts.*	*pts.*	*qrts.*	*pts.*	*gills.*
83	2	1	23	1	3
79	3	0	37	0	2
63	1	1	48	1	2
37	2	1	35	1	1
28	2	1	18	0	2
64	1	0	72	1	3

Grain, or Dry Measure.

37.			38.		
qrs.	*bush.*	*pks.*	*bush.*	*pks.*	*gals.*
73	7	3	23	3	1
68	5	2	37	2	0
97	4	2	43	3	1
83	6	3	25	2	1
37	4	2	18	2	1
46	3	1	29	3	1

Time.

39.			40.			
yrs.	*weeks.*	*days.*	D.	H.	M.	S.
37	34	5	23	17	27	35
43	29	3	37	19	36	43
56	43	6	46	18	26	54
75	24	4	84	21	47	47
84	17	3	79	16	32	36
69	14	1	41	12	17	6

Answers.

25. 3444 cwts. 0 qrs. 17 lbs.
26. 318 tons 14 cwt. 2 qrs.
27. 175 lbs. 11 oz. 2 dwts. 10 grs.
28. 40 lbs. 4 oz. 3 dwts.
29. 764 miles 7 fur. 19 per.
30. 185 per. 1 yd. 1 foot.
31. 332 yds. 3 qrs. 1 nail.
32. 410 yds. 1 qr. 1 nail.
33. 1699 acres 1 rood 6 per.
34. 158 per. 20¼ yards 7 feet.
35. 357 gallons 1 quart.
36. 236 quarts 1 pint 1 gill.
37. 408 qrs. 0 bush. 1 peck.
38. 179 bush. 1 peck 1 gall.
39. 367 years 8 weeks 1 day.
40. 314 d. 10 h. 8 m. 41 s.

Miscellaneous Exercises.

1. Bought a pair of boots for 18*s*. 6*d*., a pair of shoes for 7*s*. 6*d*., a hat for 13*s*. 8*d*., and a pair of gloves for 2*s*. 3*d*. How much did I expend altogether? . . . . Ans. £2, 1*s*. 11*d*.

2. Paid for tea 5*s*. 6*d*., for sugar 3*s*. 4*d*., for coffee 2*s*. 4*d*., for butter 3*s*. 6*d*., for cheese 5*s*. 3*d*., for beef 12*s*. 4*d*. How much did all come to? . . . . Ans. £1, 12*s*. 3*d*.

3. A shopkeeper owes the following sums:—To A, £7, 15*s*. 9½*d*.; to B, £8, 15*s*. 6*d*.; to C, £27, 13*s*. 4¼*d*.; to D, £35, 10*s*. 4*d*.; to E, £29, 3*s*. 10¼*d*.; to F, £18, 12*s*. 5*d*. What is the amount of his debts? . . . . Ans. £127, 11*s*. 3¼*d*.

4. A family is owing £3, 7*s*. 4*d*. to the baker, £4, 15*s*. 9½*d*. to the butcher, £4, 13*s*. 3½*d*. to the grocer, £1, 8*s*. 3*d*. to the shoemaker, £6, 14*s*. 8*d*. to the tailor, £8, 13*s*. 7½*d*. to the draper. How much money will it require to pay off these debts?

Ans. £29, 12*s*. 11½*d*.

5. A butcher supplied a family with 6¼ lbs. of beef on Monday, 4½ lbs. on Tuesday, 7¾ lbs. on Wednesday, 8 lbs. on Thursday, 6¾ lbs. on Friday, and 10½ lbs. on Saturday. What did the week's supply amount to? . . . . . Ans. 43¾ lbs., or 3 st. 1¾ lbs.

6. A grocer sold 7 lbs. 6 oz. tea on Monday, 9 lbs. 10 oz. on Tuesday, 5 lbs. 12 oz. on Wednesday, 14 lbs. 9 oz. on Thursday, 10 lbs. 3 oz. on Friday, and 18 lbs. 5 oz. on Saturday. How much was the week's sale? . . . . . . Ans. 65 lbs. 13 oz.

7. A tea-merchant buys 6 chests of tea. No. 1 weighs 3 qrs. 12 lbs.; No. 2, 3 qrs. 17 lbs.; No. 3, 1 cwt. 7 lbs.; No. 4, 1 cwt. 1 qr.; No. 5, 3 qrs. 23 lbs.; and No. 6, 3 qrs. 13 lbs. What is the total weight? . . . . . Ans. 5 cwts. 3 qrs. 16 lbs.

8. What is the weight of 5 hhds. sugar? No. 1 weighing 15 cwts. 3 qrs. 13 lbs.; No. 2, 13 cwts. 2 qrs. 7 lbs.; No. 3, 17 cwts. 1 qr. 18 lbs.; No. 4, 14 cwts. 3 qrs. 15 lbs.; and No. 5, 16 cwts. 1 qr. 13 lbs. . . . . Ans. 78 cwts. 0 qrs. 10 lbs.

9. A silversmith sold a gentleman silver-plate as follows:—Dishes, 7 lbs. 6 oz. 7 dwts.; plates, 5 lbs. 9 oz. 12 dwts.; spoons, 2 lbs. 6 oz. 13 dwts.; a tea-service weighing 4 lbs. 7 oz. 4 dwts. What was the total weight? . . Ans. 20 lbs. 5 oz. 16 dwts.

10. A draper bought 6 ends cloth: the first contained 37 yds. 3 qrs., 2 nails; the second, 28 yds. 2 qrs. 3 nails; the third, 43 yds. 3 qrs. 1 nail; the fourth, 37 yds. 2 qrs.; the fifth, 47 yds. 1 qr. 2 nails; the sixth, 39 yds. and 3 nails. How many yards are in all? . . . . Ans. 234 yds. 1 qr. 3 nails.

11. A hop-merchant bought 5 bags of hops, of which the first weighed 2 cwts. 3 qrs. 13 lbs.; the second, 2 cwts. 3 qrs. 11 lbs.; the third, 2 cwts. 3 qrs. 5 lbs.; the fourth, 2 cwts. 3 qrs. 12 lbs.; the fifth, 2 cwts. 3 qrs. 17 lbs. What is the weight of the whole? . . . . Ans. 14 cwts. 1 qr. 2 lbs.

COMPOUND SUBTRACTION.

Compound Subtraction is the method of deducting one Compound Number from another.

Rule.—1. Place the less number under the greater, in such a way that the numbers of the same denomination shall be below each other—pounds under pounds, shillings under shillings; and so on.

2. Beginning at the lowest denomination, subtract the under from the upper figures, and place the remainder directly below: then subtract the under from the upper figures of the next denomination; and so on with all the others, writing the remainders in each case below their respective denominations. The subtracting of the highest denomination is performed as in Simple Subtraction.

3. When the under number of any denomination is greater than the upper, add to the upper number the value of *one* of the next higher denomination, and then go on with the subtraction. As an equivalent, however, for this, the under figure of the next higher denomination must be considered as having had 1 added to it before it is deducted from the figure above, on the same principle as in Simple Subtraction.

Example.—From £36, 14*s.* 5½*d.* take £27, 17*s.* 8¾*d.*

£	*s.*	*d.*
36	14	5½
27	17	8¾
£8	16	8¾

Commencing at the lowest term, we have 3 farthings to take from 2, but as we cannot do this, we *borrow* from the pence 1 penny, or 4 farthings, which, added to the 2 farthings, make 6; then 3 from 6 and 3 remain, which are written under the farthings. We now *repay* the penny borrowed by carrying 1 to the 8 pence in the lower line; there is thus 9 pence to subtract from 5, but as this cannot be done, we borrow a shilling, or 12 pence, which, added to the 5, make 17, then 9 from 17 and 8 remains, which is placed under the pence. The *borrowed* shilling is repaid by adding 1 to the 17, making 18 to be taken from 14, but as we cannot do this, we borrow 1 pound, or 20*s.*, which being added to 14, make 34; then 18 from 34 and 16 remain, which are placed under the shillings. The £1 borrowed is carried to the £27, which are subtracted, as in Simple Subtraction.

Exercises in Money.

1.	2.	3.	4.
£35 14 5¼	£45 12 7	£315 10 6¾	£183 3 0¼
26 12 7½	33 15 6¼	276 17 8¼	97 14 5½

5.	6.	7.	8.
£276 7 3	£73 14 5	£89 13 6	£163 11 9¼
193 12 9¾	38 15 6	53 15 3¼	117 19 10½

Answers.

1. £9 1 9¾	3. £38 12 10½	5. £82 14 5¼	7. £35 18 2¾
2. 11 17 0¾	4. 85 8 6¾	6. 34 18 11	8. 45 11 10¾

9.			10.			11.			12.		
£638	17	5½	£729	12	11¾	£631	2	4½	£123	15	9¼
429	2	1¼	127	5	6½	236	11	3¾	68	18	5½

13.			14.			15.			16.		
£847	11	5½	£796	3	2¾	£1153	6	5¼	£1000	0	0
359	14	5¾	127	9	7¼	967	13	8½	9	19	9¾

Avoirdupois Weight.

17.			18.			19.					20.		
lbs.	*oz.*	*drs.*	*cwts.*	*qrs.*	*lbs.*	*tons.*	*cwts.*	*qrs.*	*lbs.*	*oz.*	*tons.*	*cwts.*	*qrs.*
17	6	13	13	1	7	14	7	2	12	0	43	10	2
12	9	15	6	2	18	8	13	1	18	6	23	16	3

Troy Weight.

21.				22.			
lbs.	*oz.*	*dwts.*	*grs.*	*lbs.*	*oz.*	*dwts.*	*grs.*
37	6	13	11	58	0	7	6
29	9	8	13	41	3	5	10

Lineal Measure.

23.			24.		
yds.	*ft.*	*in.*	*miles.*	*fur.*	*per.*
37	2	7	107	3	0
58	2	4	89	7	19

Cloth Measure.

25.			26.		
yds.	*qrs.*	*nls.*	*yds.*	*qrs.*	*nls.*
349	2	2	256	0	1
265	1	3	137	1	2

Square Measure.

27.			28.		
ac.	*ro.*	*per.*	*ac.*	*ro.*	*per.*
273	2	15	1100	0	0
78	3	23	630	1	14

Liquid Measure.

29.			30.		
gal.	*qt.*	*pt.*	*gal.*	*qt.*	*pt.*
452	2	0	191	1	0
391	3	1	69	2	1

Time.

31.				32.		
D.	H.	M.	S.	Y.	D.	H.
176	3	15	0	17	97	0
89	7	23	18	9	127	13

Answers.

9. £209 15 4¼
10. 602 7 5¼
11. 394 11 0¼
12. £54 17 3¾
13. 487 16 11¾
14. 668 13 7½
15. £185 12 8¾
16. 990 0 2¼

17. 4 lbs. 12 oz. 14 drams.
18. 6 cwts. 2 qrs. 17 lbs.
19. 5 tons 14 cwts. 21 lbs. 10 oz.
20. 19 tons 13 cwts. 3 qrs.
21. 7 lbs. 9 oz. 4 dwts. 22 grs.
22. 16 lbs. 9 oz. 1 dwt. 20 grs.
23. 20 yards 2 feet 9 inches.
24. 17 miles 3 fur. 21 per.
25. 84 yards 0 qrs. 3 nails.
26. 118 yards 2 qrs. 3 nails.
27. 194 acres 2 roods 32 per.
28. 469 acres 2 roods 26 per.
29. 60 gals. 3 quarts 1 pint.
30. 121 gals. 2 quarts 1 pint.
31. 86 days 19 h. 51 m. 42 s.
32. 7 years 334 days 11 hours.

Miscellaneous Exercises.

1. If £1750, 12*s*. 6*d*. be taken from £2000, how much will remain? . . . . Ans. £249, 7*s*. 6*d*.

2. Lent a friend £37, 15*s*., of which he has repaid me £23, 12*s*. 9*d*. How much is he still due me? . Ans. £14, 2*s*. 3*d*.

3. A merchant bought goods to the amount of £371, 15*s*. 9*d*., and sold them for £430, 8*s*. 6*d*. What profit did he make on his purchase? . . . . Ans. £58, 12*s*. 9*d*.

4. Made purchases in a grocer's shop to the amount of £3, 14*s*. 7½*d*., and gave a £5 note in payment. How much should I get back? . . . . Ans. £1, 5*s*. 4½*d*.

5. Went to market with £2, 13*s*. 6*d*. in my purse; laid out on butcher meat, 11*s*. 9*d*.; poultry, 5*s*. 3*d*.; fish, 3*s*. 7*d*.; vegetables, 1*s*. 10½*d*.; cheese, 7*s*. 8½*d*.; butter and eggs, 3*s*. 5*d*.; and on tea and sugar, 15*s*. 7*d*. How much should I have over? Ans. 4*s*. 4*d*.

6. A merchant borrowed £500, of which he repaid in April £63, 15*s*.; in June, £113, 12*s*. 8*d*.; in August, £87, 4*s*. 5*d*.; and in December, £173, 17*s*. 10*d*. How much, then, remained to be paid? . . . . Ans. £61, 10*s*. 1*d*.

7. The gross weight of a hogshead of sugar is 14 cwts. 2 qrs. 7 lbs. What is the net weight, allowing 3 qrs. 18 lbs. as the weight of the cask? . . . . Ans. 13 cwts. 2 qrs. 17 lbs.

8. A silversmith received an ingot of silver weighing 3 lbs. 8 oz. 12 dwts., of which he was to make 1 dozen table and 1 dozen tea spoons. The table-spoons weighed 1 lb. 11 oz. and 12 dwts.; the tea-spoons, 8 oz. 15 dwts. How much of the silver remained? . . . . Ans. 1 lb. 0 oz. 5 dwts.

9. From London to Newcastle, by way of York, is 276 miles 4 furlongs, and from York to Newcastle it is 82 miles 5 furlongs. How far is it from London to York? Ans. 193 miles 7 furlongs.

10. From a piece of cloth, measuring 57 yards 2 quarters, a draper sold to one person 5 yds. 3 qrs. 2 nails; to another, 7 yds. 1 qr. 3 nails; to a third, 6 yds. 2 qrs.; and to a fourth, 11 yds. 1 qr. 2 nails. How much of the piece then remained?

Ans. 26 yds. 1 qr. 1 nail.

11. A farmer laid out a piece of ground, consisting of 5 acres 2 roods 12 perches, for a fruit and vegetable garden. The vegetable-garden occupied 3 acres 1 rood 29 perches. What was the size of the fruit-garden? . . . Ans. 2 acres 0 roods 23 per.

12. A farmer brought 8 quarters 3 bushels 2 pecks of wheat to market. He sold 6 quarters 5 bushels 3 pecks. How much did he return with? . . . . . Ans. 1 qr. 5 bush. 3 pecks.

13. A wine-merchant drew off at one time 23 gals. 2 qts., and at another, 16 gals. 3 qts. 2 pts., from a cask containing 87 gals. port. What quantity then remained in the cask? Ans. 46 gals. 2 qts.

14. If it take 26 hours and 35 minutes to go to a certain town by the stage-coach, and 7 hours 45 minutes 30 seconds by railway; how much time is saved by taking the latter mode of conveyance? . . . . Ans. 18 hours 49 min. 30 sec.

COMPOUND MULTIPLICATION.

Compound Multiplication is the method of multiplying a compound number by a simple number.

RULES.

I. When the Multiplier does not exceed 12.

Rule.—1. Write the multiplier below the right-hand figures of the multiplicand.

2. Multiply the lowest denomination of the latter by the multiplier; find how many of the next higher denomination is contained in the product, and carry the number of times to that denomination, writing any remainder below the number just multiplied.

3. Multiply the next higher denomination in the same way, writing down any remainder, and carrying to the denomination above it, as before; and so on till all the denominations have been multiplied in succession: the highest denomination is multiplied as in Simple Multiplication.

Example.—Multiply £37, 15*s.* 8¾*d.* by 6.

£	*s.*	*d.*
37	15	8¾
		6
£226	14	4½

Here the multiplier, 6, is placed under the pence (the farthings being considered merely as a fraction of the pence), and beginning with the farthings we have 6 times 3, which make 18 farthings, and these being converted into pence, give 4½ pence. The ½ is placed under the farthings, and the 4 carried to the pence. Multiplying the pence, we have 6 times 8 making 48, and the 4 from the farthings make 52 pence, or 4 shillings and 4 pence. The 4*d.* is put under the pence, and the 4*s.* are carried to the shillings: then 6 times 15 are 90, and 4 are 94*s.*; that is, 4 pounds and 14 shillings over. The 14*s.* are marked under the shillings, and the £4 carried to the pounds: then multiply the £37, as in Simple Multiplication, and include the £4 from the shillings; the product, 226, is written under the pounds.

Note.—In multiplying the *shillings*, the following is the most convenient method:—First multiply the units, take the tens out of the product and carry them to the tens, writing any remainder below the units; next multiply the tens, and having added those brought from the units, halve the product; carry the half to the pounds, and if there is a remainder of 1, write 1 below the tens; if there is no remainder, there is nothing written below the tens of the shillings. The halving of the product of the tens is merely a short way of dividing it by 20, to convert it into pounds.

In the above example we first multiply the units of the shillings—6 times 5 are 30, and 4 from the pence are 34: write 4 below the units, and carrying 3 to the tens, 6 times 1 are 6, and 3 are 9; then halving this, write the remainder, 1, below the tens, and carry the half, 4, to the pounds, which are multiplied as before.

Exercises.

1.	Multiply	£0	13	7¾	by 3,	. . .	Ans.	£2	0	11¼
2.	〃	0	18	9¼	〃 5,	. . .	〃	4	13	10¼
3.	〃	1	17	6	〃 4,	. . . .	〃	7	10	0
4.	〃	1	11	9¼	〃 5,	. . .	〃	7	18	10¼
5.	〃	7	9	8½	〃 6,	. . . .	〃	44	18	3
6.	〃	39	15	5¾	〃 7,	. . .	〃	278	8	4½
7.	〃	47	12	7½	〃 12,	. . . .	〃	571	11	6
8.	〃	47	12	7½	〃 8,	. . .	〃	381	1	0
9.	〃	87	16	8¼	〃 8,	. . . .	〃	702	13	6
10.	〃	95	17	2¾	〃 6,	. . .	〃	575	3	4½
11.	〃	95	17	2¾	〃 12,	. . . .	〃	1150	6	9
12.	〃	235	3	4¼	〃 9,	. . .	〃	2116	10	2¼
13.	〃	317	17	9½	〃 10,	. . . .	〃	3178	17	11

14. 〃 3 tons 7 cwts. 2 qrs. by 7, Ans. 23 tons 12 cwts. 2 qrs.
15. 〃 173 yds. 2 ft. 7 in. 〃 8, 〃 1390 yds. 2 ft. 8 in.
16. 〃 76 days 13 h. 32 m. 〃 7, 〃 535 days 22 h. 44 m.

II. When the Multiplier is the product of two numbers, neither of which exceeds 12.

Rule.—Find the factors or numbers that produce the multiplier, then multiply the given quantity by the one factor, and the product by the other: the last product is the answer.

Examples.—Multiply £1, 2*s.* 6*d.* by 14.

£	*s.*	*d.*	
1	2	6	
		7	
7	17	6	
		2	
£15	15	0	*Ans.*

Here the multiplier, 14, is the product of the two numbers, 7 and 2; we therefore first multiply the given sum by 7, and then the product by 2.

Exercises.

1.	Multiply	£0	2	6	by 15,	. .	Ans.	£1	17	6
2.	〃	0	6	10	〃 21,	. . .	〃	7	8	6
3.	〃	0	9	1¾	〃 42,	. . .	〃	19	4	1½
4.	〃	1	14	6½	〃 63,	. . .	〃	108	16	1½
5.	〃	1	9	7	〃 81,	. . .	〃	119	16	3
6.	〃	2	13	7	〃 27,	. . .	〃	72	6	9
7.	〃	3	15	6½	〃 32,	. . .	〃	120	17	4
8.	〃	7	17	6	〃 72,	. . .	〃	567	0	0
9.	〃	8	15	4¼	〃 84,	. . .	〃	736	9	9
10.	〃	0	9	8¼	〃 132,	. . .	〃	63	18	9
11.	〃	73	19	11¾	〃 56,	. . .	〃	4143	18	10
12.	〃	96	13	9¼	〃 81,	. . .	〃	7831	15	5¼
13.	〃	67	1	3½	〃 108,	. . .	〃	7242	19	6
14.	〃	81	14	10¾	〃 144,	. . .	〃	11771	5	0

III. When the Multiplier does not exceed 144, but cannot be resolved into two factors.

Rule.—Multiply by the nearest factors, as in Rule II., then the original sum by the remainder of the multiplier, and add the two last products together for the answer.

Example.—Multiply £17, 13*s.* 6*d.* by 23.

£	*s.*	*d.*	
17	13	6	× 3
		10	
176	15	0	
		2	
353	10	0	
53	0	6	
£406	10	6	*Ans.*

Here we multiply the given sum by 10, then the product by 2, and the original sum by the remainder, 3, and add the two last products together, for the answer.

Exercises.

1. Multiply	£1	17	6¼	by	17,	Ans.	£31	17	10¼
2. 〃	2	3	9¾	〃	34,	〃	74	9	7½
3. 〃	7	9	8	〃	58,	〃	434	0	8
4. 〃	13	14	5¼	〃	65,	〃	891	18	5¼
5 〃	43	18	3½	〃	78,	〃	3425	6	9
6. 〃	3	16	8¼	〃	106,	〃	406	11	1
7. 〃	1	3	9¼	〃	123,	〃	146	3	9¾
8. 〃	14 cwts. 2 qrs. 9 lbs.			by	97,	Ans. 1414 cwts. 1 qr. 5 lbs.			
9. 〃	87 yds. 3 qrs. 2 nls.			〃	135,	〃 11863 yds. 0 qr. 2 nls.			

IV. When the Multiplier exceeds 144.

Rule.—Multiply as in Rule I.; only, instead of performing the process mentally, the figures, from their quantity, require to be written down separately, in order to be calculated.

Example.—Multiply £23, 14*s.* 7½*d.* by 374.

£	*s.*	*d.*	
23	14	7½	
		374	
£8875	9	9	*Ans.*

Here we first multiply the 2 farthings by 374, and convert them to pence, making 187*d.* and no farthings over. Carry the 187*d.* to the pence, then multiply the 7*d.* by 374, and include the 187*d.* from the farthings: the 7*d.* × 374, and adding 187, are 2805*d.*, which, converted to shillings, make 233*s.*, and 9*d.* over. Place the 9*d.* below the pence, carry 233*s.* to the shillings, then multiply the 14*s.* by 374, and include the 233*s.* from the pence, making 5469*s.*, which, converted to pounds, are £273, and 9*s.* over. Place the 9*s.* below the shillings, carry £273 to the pounds, then multiply the £23 as in Simple Multiplication, and include the £273, making £8875.

Note.—As it is generally more convenient to multiply the greater number by the less, than to multiply the less by the greater, in the above example instead of multiplying 2 farthings, &c., by 374, we multiply 374 by 2; and so on.

Exercises.

1.	Multiply	£3	7	$5\frac{1}{4}$	by 173,	. . .	Ans.	£583	6	$8\frac{1}{4}$
2.	"	7	13	$8\frac{1}{2}$	" 237,	. . .	"	1821	8	$10\frac{1}{2}$
3.	"	13	15	$9\frac{3}{4}$	" 327,	. . .	"	4509	10	$8\frac{1}{4}$
4.	"	27	18	$8\frac{1}{4}$	" 683,	. . .	"	19079	3	$6\frac{3}{4}$
5.	"	7	15	$5\frac{1}{2}$	" 751,	. . .	"	5837	9	$2\frac{1}{2}$
6.	"	6	4	$10\frac{1}{2}$	" 5346,	. . .	"	33379	1	9

7. " 13 cwts. 2 qrs. 17 lbs. by 387, Ans. 5283 cwts. 27 lbs.
8. " 37 days 9 h. 17 min. " 347, " 12973 days 5 h. 19 m.

V. When the Multiplier contains a Fraction—as $8\frac{3}{5}$.

Rule.—Multiply first by the integer or *whole* number of the multiplier, and then by the fraction (see Simple Multiplication, Rule IV., page 19), and add the products together for the answer.

Examples.—Multiply £3, 15*s.* $4\frac{1}{2}$*d.* by $8\frac{3}{5}$.

£	*s.*	*d.*
3	15	$4\frac{1}{2}$
		8
30	3	0
2	5	$2\frac{1}{2}\ \frac{4}{5}$
£32	8	$2\frac{1}{2}\ \frac{4}{5}$ *Ans.*

Here we first multiply the given sum by 8, then separately by $\frac{3}{5}$, and add the two products for the answer. For the method of *dividing* £11, 6*s.* $1\frac{1}{2}$*d.* by 5, see page 50.

	£	*s.*	*d.*
	3	15	$4\frac{1}{2} \times \frac{3}{5}$
			3
5)	11	6	$1\frac{1}{2}$
	£2	5	$2\frac{1}{2}\ \frac{4}{5}$

Exercises.

1.	Multiply	£2	16	6	by $5\frac{1}{4}$,	. . .	Ans.	£14	16	$7\frac{1}{2}$
2.	"	8	12	$4\frac{1}{2}$	" $6\frac{1}{2}$,	. . .	"	56	0	$5\frac{1}{4}$
3.	"	7	9	8	" $7\frac{3}{4}$,	. . .	"	57	19	11
4.	"	23	12	$6\frac{1}{4}$	" $8\frac{1}{6}$,	. . .	"	192	18	$11\frac{0}{4}\ \frac{1}{6}$
5.	"	36	14	$10\frac{1}{2}$	" $9\frac{4}{5}$,	. . .	"	360	1	$9\frac{1}{4}\ \frac{1}{5}$
6.	"	73	6	5	" $7\frac{5}{6}$,	. . .	"	574	6	$11\frac{0}{4}\ \frac{2}{3}$
7.	"	354	15	$7\frac{3}{4}$	" $12\frac{5}{7}$,	. . .	"	4510	16	$0\frac{3}{4}\ \frac{2}{7}$

8. " 16 cwts. 3 qrs. 12 lbs. by $27\frac{2}{5}$, Ans. 461 cwts. 3 qrs. $15\frac{1}{5}$ lbs.

Miscellaneous Exercises.

1. What do 8 yards of cloth cost, at 15*s.* 8*d.* per yard? Ans. £6, 5*s.* 4*d.*

2. What do 27 yards of linen cost, at 2*s.* $4\frac{1}{2}$*d.* per yard? Ans. £3, 4*s.* $1\frac{1}{2}$*d.*

3. Find the value of 37 yards calico, at 1*s.* 5*d.* " £2, 12*s.* 5*d.*

4. What is the value of a dozen silver spoons, weighing 1 lb. 7 oz., at 7*s.* 6*d.* per ounce? . . . . . Ans. £7, 2*s.* 6*d.*

5. What do a workman's wages amount to in 13 weeks, at 15*s.* 6*d.* per week? . . . . Ans. £10, 1*s.* 6*d.*

6. What is the yearly income of a workman who has a wage of £1, 8*s.* 6*d.* per week? . . . . . . Ans. £61, 2*s.*

7. A charitable institution gives relief to 170 individuals, to the amount of 3*s.* 3*d.* per week each. What is its yearly expenditure? . . . . Ans. £1436, 10*s.*

8. If a boy spend 4*d.* a week on sweetmeats, how much does he squander in a year? . . . . Ans. 17*s.* 4*d.*

9. If a family consume 6 loaves of bread in a week, when the price is $8\frac{1}{2}$*d.*, what is the yearly expenditure? . Ans. £11, 1*s.*

10. Find the amount of 27 cwts. of sugar, at £3, 13*s.* 6*d.* per cwt. . . . . Ans. £99, 4*s.* 6*d.*

11. A butcher bought 3 scores of ewes, for which he paid 25*s.* 9*d.* each. How much did the whole amount to? Ans. £77, 5*s.*

12. What is the cost of 71 cwts. of butter, at £4, 13*s.* $4\frac{1}{4}$*d.* per cwt.? . . . . Ans. £331, 8*s.* $1\frac{3}{4}$*d.*

13. What is the cost of 549 bags of cotton, at £12, 1*s.* $2\frac{3}{4}$*d.* per bag? . . . . Ans. £6621, 14*s.* $9\frac{3}{4}$*d.*

14. What is the cost of 936 quarters of wheat, at £2, 9*s.* $6\frac{1}{4}$*d.* per quarter? . . . . Ans. £2318, 11*s.*

15. What is the weight of 47 pieces of lead, each 25 lbs. 6 oz. 12 drs.? . . . . Ans. 1194 lbs. 13 oz. 4 drs.

16. What is the weight of 28 loaves of sugar, each 13 lbs. 6 ounces? . . . . Ans. 374 lbs. 8 oz.

17. What is the weight of 35 blocks of tin, each 2 cwts. 1 qr. 17 lbs.? . . . . Ans. 84 cwts. 0 qrs. 7 lbs.

18. What is the weight of $1\frac{1}{2}$ dozen silver table-spoons, each 2 oz. 5 dwts. and 15 grs.? . . . Ans. 41 oz. 1 dwt. 6 grs.

19. How much linen will it require to make 2 dozen shirts, each shirt requiring 3 yards 1 quarter 2 nails? Ans. 81 yards.

20. What distance will a coach travel in 29 hours, at the rate of 9 miles 6 fur. 20 per. an hour? Ans. 284 miles 4 fur. 20 per.

21. What is the value of 276 tons of iron, at £4, 16*s.* 8*d.* per ton? . . . . Ans. £1334.

22. A baker uses 4 qrs. 5 bush. 3 pecks of wheat in a week. How much does he require for a year? . Ans. 245 qrs. 3 bush.

23. A field was divided into 65 lots, each lot containing 1 rood 27 per. 15 yds. What was the size of the field?
Ans. 109 roods 27 per. 7 yds.

24. The rent of my house for a week is £2, 5*s.* $7\frac{3}{4}$*d.*, what is the rent of it for a year at the same rate? . Ans. £118, 13*s.* 7*d.*

25. There is an institution which contains 180 boys, and the yearly maintenance, clothing, and education of each comes to £18, 15*s.* $6\frac{3}{4}$*d.*; what sum of money is required to defray the expenses of the institution for a year? . Ans. £3380, 1*s.* 3*d.*

26. A farmer has a field containing 15 acres, which is sown with wheat; how much money will he receive for the crop, if each acre produces 9 qrs., and the wheat is sold at £2, 16*s.* $5\frac{1}{4}$*d.* per quarter? . . . . Ans. £380, 19*s.* $0\frac{3}{4}$*d.*

27. What is the cost of 213 sacks of oatmeal, at £1, 19*s.* $11\frac{1}{4}$*d.* per sack? . . . . Ans. £425, 6*s.* $8\frac{1}{4}$*d.*

COMPOUND DIVISION.

Compound Division is the method of dividing a compound by a simple number.

RULES.

I. When the Divisor does not exceed 12.

Rule.—Divide the highest denomination of the dividend as in Simple Division, placing the quotient below the figures divided. If there is any remainder, reduce it to the next lower denomination, add to it any of that denomination in the given quantity, and divide the amount as before: if there be again a remainder, reduce it to the next lower denomination, &c.; and so on, proceeding in this way till all the denominations have been divided. The various quotients will be of the same denomination as the dividends from which they arise.

Example.—Divide £87, 14*s.* $9\frac{3}{4}$*d.* by 7.

	£	*s.*	*d.*
7)	87	14	$9\frac{3}{4}$
	£12	10	$8\frac{1}{4}$

The 87 pounds are divided by 7; the answer is £12, and £3 over: the £3, reduced to shillings, make 60*s.*, and adding the 14*s.* in the dividend, amount to 74*s.*, which, being divided by 7, give 10*s.*, and 4*s.* over: the 4 shillings are 48 pence, to which adding the 9*d.* in the dividend, the sum is 57*d.*, and these divided by 7, give 8*d.*, and 1 penny over: a penny is 4 farthings; add to these the 3 in the dividend, making 7, which, divided by 7, give 1; that is, $\frac{1}{4}$*d.*

Exercises.

1.	Divide	£34	13	$4\frac{1}{2}$	by	2,	. . . Ans.	£17	6	$8\frac{1}{4}$
2.	"	17	12	$8\frac{1}{4}$	"	3,	. . . "	5	17	$6\frac{3}{4}$
3.	"	23	15	$8\frac{3}{4}$	"	9,	. . . "	2	12	$10\frac{1}{4}\ \frac{2}{9}$
4.	"	83	9	5	"	5,	. . . "	16	13	$10\frac{1}{2}\ \frac{2}{5}$
5.	"	57	17	$3\frac{3}{4}$	"	8,	. . . "	7	4	$7\frac{3}{4}\ \frac{7}{8}$
6.	"	93	19	$7\frac{1}{2}$	"	9,	. . . "	10	8	$10\frac{0}{4}\ \frac{2}{3}$
7.	"	124	13	$8\frac{1}{2}$	"	5,	. . . "	24	18	$8\frac{3}{4}\ \frac{3}{5}$
8.	"	137	16	4	"	10,	. . . "	13	15	$7\frac{1}{2}\ \frac{2}{5}$
9.	"	345	8	$6\frac{1}{4}$	"	11,	. . . "	31	8	$0\frac{1}{2}\ \frac{3}{11}$
10.	"	417	13	$4\frac{3}{4}$	"	12,	. . . "	34	16	$1\frac{1}{4}\ \frac{7}{12}$

11. " 85 lbs. 8 oz. 10 drs. by 8, Ans. 10 lbs. 11 oz. $1\frac{1}{4}$ drs.
12. " 136 tons 14 cwts. 2 qrs. " 9, " 15 tons 3 cwts. $3\frac{1}{3}$ qrs.
13. " 158 miles 7 fur. 26 per. " 7, " 22 miles 5 fur. $26\frac{4}{7}$ per.
14. " 76 acres 3 ro. 30 per. " 12, " 6 acres 1 ro. $25\frac{5}{6}$ per.
15. " 216 yds. 2 qrs. 3 nails " 8, " 27 yds. 0 qrs. $1\frac{3}{8}$ nail.
16. " 167 days 14 h. 34 min. " 7, " 23 days 22 h. $39\frac{1}{7}$ m.

II. When the Divisor exceeds 12.

Rule.—Divide as in Rule I., only employ *long* instead of short division.

Example.—Divide £484, 19*s.* 7$\frac{3}{4}$*d.* by 73.

```
    £    s.   d.   £  s.  d.
73)484  19  7¾(6  12  10¼ 62/73 Ans.
   438
    46
    20
   939(12
   73
   209
   146
    63
    12
   763(10
   73
    33
     4
   135(1
    73
    62 = 62/73
```

The pounds are first divided by 73; the answer is £6, and £46 over: the £46 are reduced to shillings, by multiplying by 20, and the 19*s.* in the dividend being added, make together 939 shillings, which, divided by 73, give 12*s.*, and 63*s.* over: the 63 shillings are reduced to pence, and the 7*d.* being added, make 763*d.*, which, divided by 73, give 10*d.*, and 33 pence over: the 33*d.* being reduced to farthings, make, with the addition of the 3 farthings in the sum, 135, and these, divided by 73, give $\frac{1}{4}$*d.*, and 62 over. The whole answer is thus £6, 12*s.* 10$\frac{1}{4}$*d.* $\frac{62}{73}$.

Exercises.

1.	Divide	£46	13	6	by	17,	Ans.	£2	14	10$\frac{3}{4}$ $\frac{13}{17}$
2.	"	97	8	4$\frac{1}{2}$	"	23,	"	4	4	8$\frac{1}{2}$ $\frac{4}{23}$
3.	"	130	10	3$\frac{1}{4}$	"	31,	"	4	4	2$\frac{1}{4}$ $\frac{22}{31}$
4.	"	271	15	8	"	47,	"	5	15	7$\frac{3}{4}$ $\frac{15}{47}$
5.	"	3762	10	5	"	59,	"	63	15	5$\frac{0}{4}$ $\frac{40}{59}$
6.	"	4108	9	2$\frac{3}{4}$	"	67,	"	61	6	4$\frac{3}{4}$ $\frac{34}{67}$
7.	"	1386	12	8	"	93,	"	14	18	2$\frac{1}{4}$ $\frac{59}{93}$
8.	"	7329	16	5$\frac{1}{2}$	"	842,	"	8	14	1$\frac{1}{4}$ $\frac{36}{842}$
9.	"	8456	12	3$\frac{1}{4}$	"	573,	"	14	15	2$\frac{0}{4}$ $\frac{86}{573}$
10.	"	5678	19	11$\frac{1}{2}$	"	125,	"	45	8	7$\frac{1}{2}$ $\frac{88}{125}$
11.	"	10563	14	5	"	137,	"	77	2	1$\frac{3}{4}$ $\frac{21}{137}$
12.	"	31476	5	0$\frac{3}{4}$	"	261,	"	120	11	11$\frac{1}{2}$ $\frac{189}{261}$
13.	"	27395	17	6	"	376,	"	72	17	2$\frac{1}{2}$ $\frac{344}{376}$
14.	"	17841	0	8	"	537,	"	33	4	5$\frac{1}{2}$ $\frac{314}{537}$
15.	"	16580	14	10$\frac{1}{4}$	"	643,	"	25	15	8$\frac{3}{4}$ $\frac{48}{643}$

16. " 367 tons 15 cwts. 3 qrs. by 87, Ans. 4 tons 4 cwts. 2$\frac{17}{87}$ qrs.
17. " 15376 ac. 2 ro. 15 per. " 285, " 53 ac. 3 ro. 32$\frac{135}{285}$ per.
18. " 4383 yds. 3 qrs. " 203, " 21 yds. 2 qrs. 1$\frac{106}{203}$ nls.
19. " 3769 qrs. 7 bush. 3 pks. " 257, " 14 qrs. 5 b. 1$\frac{106}{257}$ pks.
20. " 7610 gals. 3 qts. 1 pt. " 371, " 20 gals. 2 qts. 0$\frac{43}{371}$ pt.

NOTE.—WHEN THE DIVISOR is the product of two numbers, neither of which exceeds 12, *short* division may be employed, by first dividing the given sum by the one number, and the quotient thus obtained, by the other: the last quotient is the answer.

Example.—Divide £376, 11*s.* $1\frac{1}{2}$*d.* by 63.

	£	*s.*	*d.*	
7)	376	11	$1\frac{1}{2}$	
9)	53	15	$10\frac{1}{2}$	
	£5	19	$6\frac{1}{2}$	*Ans.*

As 63 is the product of 7 and 9, the given sum is divided by 7, and the quotient, £53, 15*s.* $10\frac{1}{2}$*d.*, by 9: the last quotient is the answer.

Exercises.

1.	Divide £	85	13	$5\frac{1}{2}$	by 16,	. . .	Ans. £	5	7	$1\frac{0}{4}\ \frac{3}{8}$
2.	"	65	15	$3\frac{1}{4}$	" 21,	. . .	"	3	2	$7\frac{1}{2}\ \frac{1}{3}$
3.	"	134	7	$4\frac{1}{2}$	" 27,	. . . .	"	4	19	$6\frac{1}{4}\ \frac{5}{9}$
4.	"	247	18	3	" 35,	. . .	"	7	1	$7\frac{3}{4}\ \frac{31}{35}$
5.	"	293	14	$8\frac{1}{2}$	" 42,	. . . .	"	6	19	$10\frac{1}{4}\ \frac{20}{21}$
6.	"	853	12	$8\frac{1}{2}$	" 96,	. . .	"	8	17	$10\frac{0}{4}\ \frac{34}{96}$

III. WHEN THE DIVISOR IS 10, 100, 1000, OR 1 WITH ANY OTHER NUMBER OF NOTHINGS ANNEXED.

RULE.—Point off as many figures from the *right* of the highest denomination of the dividend, as there are nothings in the divisor; the remaining figures are the quotient of the denomination divided: reduce the figures pointed off to the next lower denomination, and add any of the same denomination in the given sum; then point off as before for a further quotient, and reduce the figures pointed off to the next lower denomination; and so on. The figures that remain at each stage, after the pointing off, form the answer required.

Example.—Divide £3642, 15*s.* 6*d.* by 100.

```
 £     s.  d.
36,42  15   6
  20
 8,55
  12
 6,66
   4
 2,64  Ans. £36 8 6½ 64/100
```

Here there being two nothings in the divisor, 2 figures are pointed off, from the *right* of the dividend, at each stage of the process. The figures that remain after the pointings off are —£36, then 8*s.*, then 6*d.*, and 2 farthings, with a remainder of 64, and form the answer.

Exercises.

1.	Divide £	496	17	$8\frac{1}{4}$	by 10,	. .	Ans. £	49	13	$9\frac{1}{4}$
2.	"	879	13	$11\frac{3}{4}$	" 10,	.	"	87	19	$4\frac{3}{4}\ \frac{1}{10}$
3.	"	2984	12	$7\frac{3}{4}$	" 100,	. . .	"	29	16	$11\frac{0}{4}\ \frac{47}{100}$
4.	"	46395	9	$8\frac{1}{4}$	" 100,	.	"	463	19	$1\frac{0}{4}\ \frac{66}{100}$
5.	"	98400	19	$7\frac{1}{4}$	" 100,	. . .	"	984	0	$2\frac{1}{4}\ \frac{42}{100}$
6.	"	378421	18	11	" 1000,	.	"	378	8	$5\frac{1}{4}\ \frac{88}{1000}$

IV. When the Divisor contains a Fraction—as $3\frac{3}{5}$.

Rule.—Multiply the integer or *whole* number of the divisor, by the under figure of the fraction, adding the upper figure to the product: multiply the dividend also by the under figure of the fraction: then divide the one product by the other.

Example.—Divide £82, 15*s.* 8*d.* by $5\frac{3}{4}$*d.*

	£	s.	d.				
$5\frac{3}{4}$)	82	15	8				
4			4	£	s.	d.	
23)	331	2	8(	14	7	$11\frac{1}{4}\frac{5}{23}$	*Ans.*

Here the integer of the divisor is multiplied by 4, the under figure of the fraction, and 3, the upper figure, is added to the product, making 23 for the divisor; the multiplicand is also multiplied by 4, making £331, 2*s.* 8*d.*, the division is then proceeded with as in Rule II.

Exercises.

1.	Divide	£36	13	$5\frac{1}{4}$	by	$3\frac{1}{2}$,	Ans. £10	9	$6\frac{1}{2}\frac{4}{7}$
2.	"	58	6	$7\frac{3}{4}$	"	$4\frac{1}{4}$,	" 13	14	$6\frac{0}{4}\frac{4}{17}$
3.	"	87	12	$10\frac{1}{2}$	"	$6\frac{3}{4}$,	" 12	19	$8\frac{0}{4}\frac{8}{9}$
4.	"	71	4	8	"	$7\frac{2}{3}$,	" 9	5	$9\frac{3}{4}\frac{16}{23}$
5.	"	138	11	$3\frac{3}{4}$	"	$12\frac{3}{7}$,	" 11	2	$11\frac{3}{4}$
6.	"	754	10	6	"	$26\frac{4}{5}$,	" 28	3	$0\frac{3}{4}\frac{51}{67}$

V. When the Divisor is a Compound Number.

Rule.—Reduce both divisor and dividend to the lowest denomination that is in the one or the other; thus, if the lowest denomination in either be pence, reduce both to pence. Having now two simple numbers, proceed by long or short division, as the case may require.

Example.—Divide £58, 18*s.* $8\frac{1}{4}$*d.* by £2, 6*s.* 3*d.*

£ s. d.	£	s.	d.
2 6 3	58	18	$8\frac{1}{4}$
20	20		
46	1178		
12	12		
555	14144		
4	4		
2220)	56577(	$25\frac{1077}{2220}$	*Ans.*

Here the divisor and the dividend are both reduced to farthings, and then the one sum is divided by the other, by Rule II.

Exercises.

1.	Divide	£3 16 6 by £0 4 6,	Ans.	17
2.	"	15 6 8 " 0 7 4,	"	$41\frac{9}{11}$
3.	"	56 18 0 " 1 17 4,	"	$30\frac{27}{56}$
4.	"	375 14 4 " 23 15 0,	"	$15\frac{1168}{1425}$
5.	"	85 cwts. 2 qrs. 14 lbs. by 3 qrs. 7 lbs.	"	$105\frac{5}{13}$
6.	"	97 yds. 3 qrs. " 3 yds. 2 qrs.	"	$27\frac{13}{14}$

Miscellaneous Exercises in Compound Division.

1. A gentleman gives £10, 10*s*. to be equally divided among 30 poor people. What is the share of each? . . . Ans. 7*s*.

2. A captain of a vessel receives £276, 15*s*. 8*d*. to be equally divided among his crew, consisting of 45 men. What is each man's share? . . . . Ans. £6, 3*s*. $0\frac{0}{4}$*d*. $\frac{32}{45}$

3. Paid £3, 17*s*. 6*d*. for 30 yards of linen. What is the price per yard? . . . . Ans. 2*s*. 7*d*.

4. What is the price of sugar per lb. when it is 77*s*. per cwt.? . . . . Ans. $8\frac{1}{4}$*d*.

5. If the distance between Edinburgh and Newcastle, which is 118 miles 6 fur. 25 poles, be divided into 12 stages of equal length, what is the length of each stage? . Ans. 9 miles 7 fur. $8\frac{3}{4}$ poles.

6. How many parcels of coffee, of 7 lbs. each, may be made out of a cask containing 3 cwts. 1 qr. 14 lbs.? . Ans. 54 parcels.

7. How many yards of cloth, at 9*s*. 4*d*. may be bought for £12, 12*s*.? . . . . Ans. 27 yards.

8. 1 dozen silver table-spoons weighed 1 lb. 10 oz. 12 dwts. What is the weight of each? . Ans. 1 oz. 17 dwts. 16 grains.

9. 25 bales of cotton weighed 82 cwts. 3 qrs. 15 lbs. What is the average weight of each bale? . Ans. 3 cwts. 1 qr. $7\frac{8}{25}$ lbs.

10. How many shirts may be made out of 136 yds. 2 qrs. linen, each shirt requiring 3 yds. 2 qrs.? . . . Ans. 39 shirts.

11. 26 yards of cloth cost £18, 14*s*. What is the price per yard? . . . . Ans. 14*s*. $4\frac{1}{2}$*d*. $\frac{6}{13}$

12. What is the price of 1 yard of linen, when a piece containing $25\frac{1}{2}$ yards cost £3, 8*s*. 9*d*.? . . . . Ans. 2*s*. $8\frac{1}{4}$*d*. $\frac{7}{17}$

13. When iron is £6, 16*s*. 10*d*. per ton, what is the value of 1 cwt.? . . . . Ans. 6*s*. $10\frac{0}{4}$*d*. $\frac{2}{5}$

14. What is the price of an article per lb. at the rate of £5, 2*s*. 8*d*. per cwt.? . . . . Ans. 11*d*.

15. What is the price of a bushel of wheat, at the rate of £3, 18*s*. 8*d*. per quarter? . . . . . . Ans. 9*s*. 10*d*.

16. Among how many persons may £576, 19*s*. 6*d*. be divided, to give each £3, 13*s*. 6*d*.? . . . . . Ans. 157 persons.

17. A farm containing 237 acres is let for £325, 15*s*. How much does it pay per acre? . . . . Ans. £1, 7*s*. $5\frac{3}{4}$*d*. $\frac{117}{237}$

18. If a pound of gold is coined into 47 sovereigns, what is the weight of each sovereign? . . . . . . Ans. $5\frac{5}{47}$ dwts.

19. A tradesman's yearly income is £176. How much may he spend weekly, in order to save £25 per year? Ans. £2, 18*s*. $0\frac{3}{4}$ $\frac{9}{13}$

20. The sum of £738, 15*s*. is to be made up by 463 persons. How much must each pay? . . . . Ans. £1, 11*s*. $10\frac{3}{4}$*d*. $\frac{347}{463}$

21. If 112 ingots of gold are worth £77878, 5*s*. 4*d*., what is the value of one? . . . . Ans. £695, 6*s*. 10*d*.

22. Prize-money to the amount of £683, 5*s*. 4*d*. is to be equally divided among 83 seamen, how much will each receive?

Ans. £8, 4*s*. $7\frac{1}{2}$*d*. $\frac{70}{83}$

Miscellaneous Exercises in Compound Addition, Subtraction, Multiplication, and Division.

1. A purchases from B a hogshead of sugar, of which the value was £73, 10*s.*; a box of tea, £54, 16*s.* 8*d.*; and a pipe of wine, £93, 5*s.* In return, B receives from A £50 in money, and 3 pieces of cloth valued at £67, 15*s.* How much does A still owe B?

Ans. £103, 16*s.* 8*d.*

2. A lady went to market with £5, 3*s.* 11*d.*, and laid out on groceries 18*s.* $4\frac{1}{2}$*d.*; on bread, 12*s.* $5\frac{1}{4}$*d.*; on beef, £1, 6*s.* 3*d.*; and on various other articles, 4*s.* 11*d.* How much money should she have remaining? . . . . . . Ans. £2, 1*s.* $11\frac{1}{4}$*d.*

3. A gentleman's income is £1200 a year, and he spends on an average, £1, 17*s.* $5\frac{3}{4}$*d.* every day. How much does he save in a year—a week—a day? . . . Ans. Yearly, £516, 0*s.* $1\frac{1}{4}$*d.*; Weekly, £9, 18*s.* $5\frac{1}{2}$*d.* $\frac{13}{25}$; Daily, £1, 8*s.* $3\frac{1}{4}$*d.* $\frac{60}{365}$.

4. A gentleman's income is £460 a year; he wishes to save £53, 19*s.* 6*d.* annually. How much may he spend a week?—per day? . Ans. Weekly, £7, 16*s.* $1\frac{3}{4}$*d.* $\frac{44}{52}$; Daily, £1, 2*s.* $2\frac{3}{4}$*d.* $\frac{329}{365}$.

5. A farmer sold 39 quarters of wheat, at £2, 16*s.* 11*d.* per quarter, and 23 quarters, at £2, 4*s.* $7\frac{3}{4}$*d.* How much money did he receive?—and what was the average price per quarter?

Ans. £162, 6*s.* $7\frac{1}{4}$*d.*; average price, £2, 12*s.* $4\frac{1}{4}$*d.* $\frac{31}{62}$.

6. A manufacturer sold 739 yards of calico, at $5\frac{3}{4}$*d.* per yard; 69 yards of tartan, at 4*s.* $7\frac{1}{2}$*d.* per yard; 73 yards of carpeting, at 3*s.* 9*d.* per yard. How much money did he draw?

Ans. £47, 6*s.* $11\frac{3}{4}$*d.*

7. A bricklayer's wages are 3*s.* $10\frac{1}{2}$*d.* per day; he requires to save £23 for house-rent and clothes. How much may he spend per day? . . . . Ans. 2*s.* $7\frac{1}{4}$*d.* $\frac{185}{365}$.

8. If a workman gain 23*s.* 7*d.* per week, and spend 18*s.* $3\frac{1}{2}$*d.* per week, how much does he save in a year? Ans. £13, 15*s.* 2*d.*

9. A gentleman's annual income is £1000, and his daily expenses are £1, 17*s.* $3\frac{1}{2}$*d.* How much does he save in 9 years? . . . . Ans. £2874, 16*s.* $10\frac{1}{2}$*d.*

10. If 1869 sovereigns are coined from forty troy pounds of gold, what is the weight of a sovereign? Ans. 5 dwts. $3\frac{613}{1869}$ gr.

11. A purse contains £95, 14 crowns, 23 half-crowns, 19 shillings, 15 sixpences, and 11 fourpenny-pieces, and the money is equally divided among 12 individuals. How much does each receive? . . . . Ans. £8, 11*s.* $5\frac{1}{2}$*d.* $\frac{8}{12}$.

12. A tradesman being insolvent, called all his creditors together, and found he owed to A £53, 7*s.* 6*d.*; to B, £105, 10*s.*; to C, £34, 5*s.* 2*d.*; to D, £28, 16*s.* 5*d.*; to E, £14, 15*s.* 8*d.*; to F, £112, 9*s.*; to G, £143, 12*s.* 9*d.* The value of his stock was £212, 6*s.*; the debts due to him amounted to £112, 8*s.* 3*d.*, besides £21, 10*s.* 5*d.* money in hand. How much would his creditors lose by taking the whole of his effects? . Ans. £146, 11*s.* 10*d.*

SIMPLE PROPORTION.

Proportion is the method of finding an *unknown* number, by means of certain other given numbers to which it bears a proportion.

Simple Proportion is where three numbers, or terms, as they are called, are given in order to find a fourth; hence it is also called the Rule of Three.

Compound Proportion, as afterwards explained, is where more than three numbers or terms (usually five) are given to find out the unknown quantity.

In Simple Proportion, two out of the three given numbers are always of the same *kind*—as, for instance, 9 yards and 18 yards; and the third is the same in kind as the fourth number sought—thus, if the third were pounds, the fourth would also be pounds.

RULES.

I. Rule for Stating.*—1. Write down as the *third* of the three terms (which are all to be placed in one line) that term which is of the same kind as the answer sought.

2. Consider, from the nature of the question, whether the answer should be greater or less than the third term: if greater, place the *greater* of the other two terms second; if less, place the *less* second; and the remaining term in each case, first. Two dots, thus [:] are placed between the first and second term, and four dots [::] between the second and the third; the three terms when stated appear thus—3 : 6 :: 12.

II. Rule for Working.—1. Reduce the first and second terms, if compound, to the same simple denomination: reduce also the third term, if compound, to its lowest given denomination.

2. Multiply the second and third terms together, and divide their product by the first. The quotient will be the answer sought, and is always of *the same kind as the third term.* In multiplying, the second is placed under the third term, or the third under the second, according to convenience.

3. Convert the answer, when necessary, into its highest denomination; thus, if the answer were in pence, it must be converted into pounds.

Note.—When the second term does not exceed 12, it will be more convenient to multiply the third term as in Compound Multiplication, than to reduce it to its lowest denomination, before multiplying. See Example 1, page 58.

For an explanation of the reason of the rule, see page 68.

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* See another mode of stating the rule, page 59.

Example 1.—If 9 lbs. of tea cost 27*s.*, how many lbs. may be purchased for 18*s.*? State and work the question as follows:—

```
 s.   s.    lbs.
27 : 18 :: 9
           9
27 ) 162
Ans.   6 lbs.
```

Here the answer wanted is lbs.: the 9 lbs., therefore, are made the third term; and since a less quantity will be bought for 18*s.* than for 27*s.*, the answer will be less than the third term; therefore 18*s.*, the lesser of the other two terms, is made the second, and 27*s.*, the greater, the first: then multiplying the second and third terms together, we have 162, which, divided by 27, give 6 lbs. for the fourth or previously unknown term.

Example 2.—If 12 men can execute a piece of work in 18 days, how long will 3 men take to do the same?

```
Men. Men.  Days.
 3 : 12 :: 18
           12
        3)216
     Ans.  72 days.
```

In this example the answer wanted is days, the 18 days are therefore made the third term; and as 3 men will take longer than 12 men to execute the work, and the answer should consequently be greater than the third term, the greater of the other two is placed second, and the lesser first.

Example 3.—If 1 cwt. 1 qr. 6 lbs. of sugar cost £3, 6*s.* 11*d.*, what will 2 cwts. 3 qrs. 5 lbs. cost?

```
cwts. qrs. lbs.    cwts. qrs. lbs.    £   s.   d.
 1    1    6    :   2    3    5   ::  3   6   11
 4                  4                20
 5                 11                66
28                 28                12
146                93               803
                  22
                  313
                  803
                  939
                2504       12
                251339 ( 1721½
                146    2,0 ) 14,3  5½
                1053        £7  3  5½ Ans.
                1022
                  313
                  292
                  219
                  146
                   73
                    4
                  292(2
                  292
```

Here, in order to make the first term a simple number, it is reduced to its lowest denomination, namely, lbs.: the second term is therefore reduced to lbs. also, that both may be alike. The third term is reduced to its lowest denomination, pence. After multiplying and dividing, according to rule, the quotient is 1721 pence, and a remainder of 73*d.*: these being further reduced to farthings, and divided by the first term, give 2 farthings, which are placed in the quotient: then convert the 1721½*d.* into pounds, &c., and the answer is £7, 3*s.* 5½*d.*

CONTRACTION OF THIS RULE.—The working of the questions may often be much shortened as follows:—

Divide the two first terms, or the first and last terms (but never the second and third) by any number that will divide both without a remainder; then proceed according to the rule, with the quotients, instead of the original terms which are now said to be cancelled—a stroke being drawn across them to indicate this.

The cancelling does not alter the relative proportion between the two terms, because both have been divided by the same number, and the answer to the question will be the same as if the original terms had been employed; whilst, from the terms having been lessened, the calculations are more easily performed.

Example 1.—If 15 yards cost £9, 12*s.*, what will 55 yards cost?

yds.		*yds.*		*£*	*s.*	*d.*	
~~15~~	:	~~55~~	::	9	12	0	
3		11				11	
			3)	105	12	0	
				£35	4	0	*Ans.*

Here the first two terms are divided by 5, and the quotients employed instead of the original terms, which are cancelled.

See Note as to the multiplication of the third term, p. 56.

Example 2.—If 5 lbs. cost £1, 7*s.* 6*d.*, what will 45 lbs. cost?

lbs.		*lbs.*		*£*	*s.*	*d.*	
~~5~~	:	~~45~~	::	1	7	6	
		9				9	
				£12	7	6	*Ans.*

Here the first two terms are divided by 5; and the *first* being thus *entirely* cancelled, all that is necessary is to *multiply* the third term by 9, the cancel of the second.

Example 3.—If 34 yds. cost £4, 8*s.*, how much will 11*s.* purchase?

£	*s.*		*s.*		*yds.*	
4	8	:	~~11~~	::	34	
20						
~~88~~				8)	34	
8					$4\frac{1}{4}$	*yds. Ans.*

Here the first two terms are divided by 11; and the *second* being thus entirely cancelled, it is only necessary to *divide* the third term by 8, the cancel of the first.

Example 4.—If 7 yards cost 28*s.*, how much will £5, 3*s.* purchase?

s.		*£*	*s.*		*yds*
~~28~~	:	5	3	::	~~7~~
4		20			
	4)	103			
Ans.		$25\frac{3}{4}$ *yds.*			

Here the first and last terms are divided by 7; and the *last* being entirely cancelled, the second is divided by 4, the cancel of the first.

Example 5.—If 7 yards cost 28*s.*, what will 26 yards cost?

yds.		*yds.*		*s.*
~~7~~	:	26	::	~~28~~
		4		4
		104 = £5, 4*s.* *Ans.*		

Here the first and last terms are divided by 7; and the *first* being entirely cancelled, 26 is multiplied by 4, the cancel of the last.

ANOTHER METHOD OF STATING THE TERMS IN PROPORTION.—Instead of stating the terms according to the rule, as given at page 56, it will often be found simpler to state them in the following order—the *working* of the question being the same as before :—

I. When the greater of the two similar terms requires the greater answer, or the lesser requires the less answer.

Rule.—Write down the terms in the order in which the sense can be plainly expressed in words, making that the *middle* term which is of the same kind as the answer required: then, as before, multiply the two last terms together, and divide the product by the first. The answer will be of the same kind as the *middle* term.

The terms may be *cancelled*, when practicable, as already explained.

Example.—If 10 yards of cotton cost 20*s.*, what will 30 yards cost?

```
yds.    s.     yds.
 10  :  20  ::  30
                20
            10)600
             60s. = £3  Ans.
```

Here the terms are stated in the same order as they are expressed in words: they might have been cancelled by striking out the nothings in the first and last terms.

The reason for multiplying and dividing is as follows:—The price of 10 yards being 20*s.*, if we divide 20 by 10, we will get the price of 1 yard—namely, 2*s.*: if we then multiply the price of *one* yard by 30, we will get the price of 30 yards—namely, 60*s.* In practice, we multiply before dividing, as it is usually more convenient.

II. When the greater of the two similar terms requires a *less* answer, or the less requires a *greater* answer.

Rule.—State the two similar terms in the *inverse* order in which they are stated by Rule I. Thus—first suppose the question to be stated according to the meaning, as in Rule I., then reverse the terms, placing the first as the third, and the third as the first.

Example.—If 12 men take 18 days to execute a piece of work, how many days will 3 men take?

```
Men.  Days.  Men.
 3  :  18  ::  12
       12
    3)216
 Ans. 72 days.
```

By Rule I., this would be stated—12 : 18 :: 3; but as the smaller number of men, 3, will require the *greater* answer, the 3 is placed first, and 12 last.

Exercises.

1. If 4 yds. of linen cost 10*s.*, what will 16 yds. cost? Ans. £2.
2. If 6 lbs. of tea cost 26*s.* what will 28 lbs. cost? Ans. £6, 1*s.* 4*d.*
3. I paid 8*d.* for 2 oz. of tea; what is that per lb.? " 5*s.* 4*d.*
4. If 15 yards of cloth cost £10, what is the value of 3 yards? . . . . Ans. £2.
5. If for 10*s.* I can buy 4 yards of linen, how much can I purchase for 40*s.*? . . . . Ans. 16 yards.
6. If I pay 20*s.* for 28 lbs. of sugar, how much may be had for 5*s.*? . . . . Ans. 7 lbs.
7. If 10 men can do a piece of work in 8 days, how long will 4 men take to do the same? . . . . . Ans. 20 days.
8. If a piece of work can be done in 16 days by 6 men, how many men must be employed to do it in 4 days? Ans. 24 men.

9. If 45 yards of cloth cost £18, 14*s.* 6*d.*, what will 9 yards cost? . . . . Ans. £3, 14*s.* 10$\frac{3}{4}$*d.* $\frac{1}{5}$

10. If 8 oz. of silver-plate cost £2, 16*s.*, what will 3 lbs. 4 oz. cost? . . . . Ans. £14.

11. A lady went to a china-shop and purchased 19 dinner-plates, for which she paid 9*s.* 6*d.*; at what rate per dozen did she buy the plates? . . . . Ans. 6*s.*

12. A gentleman paid an account for 2300 feet of gas, supplied by meter, amounting to 19*s.* 6*d.*; at what rate, per 1000 feet was the gas charged? . . . . Ans. 8*s.* 5$\frac{1}{2}$*d.* $\frac{22}{23}$

13. What does a servant's wages amount to in 13 weeks, at the rate of £25 a year? . . . . Ans. £6, 5*s.*

14. How much cloth, at 12*s.* 6*d.* per yard, should be given in exchange for 85 lbs. of tea, at 5*s.* 4*d.* per lb.? Ans. 36$\frac{4}{15}$ yards.

15. Bought 13 cwts. 3 qrs. of flax, at £65 per ton; what did it amount to? . . . . Ans. £44, 13*s.* 9*d.*

16. What will 3 cwts. 2 qrs. 7 lbs. of butter amount to, at 12*s.* 6*d.* per stone of 14 lbs.? . . . . Ans. £17, 16*s.* 3*d.*

17. If the yearly rent of a farm of 276 acres be £320, what should be the rent of 63 acres? . . . Ans. £73, 0*s.* 10$\frac{1}{4}$*d.* $\frac{17}{23}$

18. If 36 reams of paper cost £28, 15*s.*, how much will 150 reams cost? . . . . Ans. £119, 15*s.* 10*d.*

19. Find the value of 13 cwts. 3 qrs. 18 lbs. of sugar, at 76*s.* 8*d.* per cwt., ... Ans. £53, 6*s.* 5$\frac{3}{4}$*d.* $\frac{3}{7}$

20. A piece of blue cloth, measuring 65 yards, cost £43, 15*s.*; what is the value of 3$\frac{1}{4}$ yards? . . . . . Ans. £2, 3*s.* 9*d.*

21. If 10 yards of linen be required for 3 shirts, how many will a piece containing 53$\frac{1}{2}$ yards make? . . . Ans. 16$\frac{1}{20}$ shirts.

22. How much linen will be required for 18 shirts, if 3 require 10$\frac{1}{2}$ yards? . . . . Ans. 63 yards.

23. How much will 4 casks of butter cost, each weighing 44 lbs., at the rate of 12*s.* 6*d.* per stone of 14 lbs.? Ans. £7, 17*s.* 1$\frac{1}{2}$*d.* $\frac{6}{7}$

24. A piece of cloth 8 yards long cost £4, 16*s.*, how much at that rate would a piece 42 yards long cost? . . Ans. £25, 4*s.*

25. Paid 8*s.* 9*d.* for dyeing a piece of silk 7 yards long; how much will a piece 26 yards long cost at the same rate?

Ans. £1, 12*s.* 6*d.*

26. If it require 5 reams of paper to print 100 copies of a book, how much will be wanted for an edition of 3500 copies?

Ans. 175 reams.

27. If I pay £5, 15*s.* for a piece of cloth measuring 20 yards, how much will 3 yards cost? . . . . . . Ans. 17*s.* 3*d.*

28. If 6 silver tea-spoons weigh 4$\frac{1}{2}$ oz., how much silver will there be in 3 doz.? . . . . Ans. 2 lbs. 3 oz.

29. A ship is provisioned for 60 days, allowing each man 3 lbs. a day; how much should be allowed to make these provisions last 80 days? . . . . Ans. 2$\frac{1}{4}$ lbs.

30. If 18 persons can reap a field in 10 days, how many must be employed to do it in 4 days? . . . . Ans. 45 persons.

31. If $5\frac{1}{4}$ yards of cloth cost £3, 17*s.* 6*d.*, what will $12\frac{1}{2}$ yards cost? . . . . Ans. £9, 4*s.* $6\frac{1}{4}$*d.* $\frac{1}{7}$

32. If I pay £1, 18*s.* for $2\frac{3}{8}$ yards of cloth, how much can I purchase for £5, 14*s.*? . . . . . . Ans. $7\frac{1}{8}$ yards.

33. What should be paid for 14 lbs. of sugar, at the rate of £3, 14*s.* 8*d.* per cwt.? . . . . Ans. 9*s.* 4*d.*

34. If a piece of muslin, measuring 16 yards, cost 26*s.* 8*d.*, how much should 6 yards of it cost? . . . . . Ans. 10*s.*

35. What will 9 cwts. 2 qrs. 15 lbs. of sugar amount to, at the rate of 35*s.* 6*d.* per 50 lbs.? . . . . Ans. £38, 6*s.* $1\frac{0}{4}$*d.* $\frac{8}{25}$

36. Find the price of 16 lbs. of butter, at the rate of £4, 17*s.* 6*d.* per cwt., ... Ans. 13*s.* $11\frac{0}{4}$*d.* $\frac{4}{7}$

37. What is the price of 12 lbs. of coffee, when a cwt. cost £9, 15*s.*? . . . . Ans. £1, 0*s.* $10\frac{1}{2}$*d.* $\frac{6}{7}$

38. Find the cost of a cheese, weighing $24\frac{1}{2}$ lbs., at £3, 6*s.* 8*d.* per cwt., ... Ans. 14*s.* 7*d.*

39. A hogshead of sugar, weighing net 13 cwts. 1 qr. 17 lbs., amounted to £48, 7*s.* 7*d.*; what is the price per cwt.?
Ans. £3, 12*s.* $2\frac{1}{4}$*d.* $\frac{763}{1501}$

40. What is the value of 3 cwts. 1 qr. 18 lbs. of flax, at £73, 15*s.* per ton? . . . . Ans. £12, 11*s.* $6\frac{1}{4}$*d.* $\frac{13}{14}$

41. What will a piece of cloth, measuring 27 yds. 3 qrs. amount to, if 2 yds. 1 qr. 3 nails of the same cost £1, 12*s.* 8*d.*?
Ans. £18, 11*s.* $10\frac{3}{4}$*d.* $\frac{1}{13}$

42. Find the value of 10 qrs. 5 bushels of wheat, at 8*s.* 9*d.* per bushel, ... Ans. £37, 3*s.* 9*d.*

43. How much wheat may be bought for £100, when the price is 72*s.* 6*d.* per quarter? . . . . . Ans. $27\frac{17}{29}$ quarters.

44. Bought a lot of tobacco, weighing 87 cwts. 3 qrs. 22 lbs., for £576, 17*s.* 8*d.*; what did it cost per cwt.? Ans. £6, 11*s.* $2\frac{1}{4}$*d.* $\frac{523}{4925}$

45. If I pay 21*s.* 8*d.* for 5 lbs. of tea, what quantity at the same rate will come to £18, 19*s.* 4*d.*? . . . . . Ans. $87\frac{7}{13}$ lbs.

46. In sowing a field with wheat, it was found that it required 2 pecks for every 3 roods; how much did it take for the whole field, which measured 7 acres 2 roods 30 poles? Ans $20\frac{1}{2}$ pecks.

47. Bought a piece of linen, containing 54 yards, for £5, 16*s.* 8*d.*, gave a friend at his request 16 yards; what should he pay me for that quantity? . . . . Ans. £1, 14*s.* $6\frac{3}{4}$*d.* $\frac{7}{27}$

48. If 14 lbs. of iron cost 3*s.* $7\frac{1}{2}$*d.*, how much iron may I purchase for £21, 4*s.* $1\frac{1}{2}$*d.*? . . . . . Ans. 14 cwts. 2 qrs. 14 lbs.

49. What is the price of 3 pieces of scarlet, each 57 yards, when 76 yards cost £72, 3*s.* $4\frac{1}{2}$*d.*? . . . . Ans. £162, 7*s.* $6\frac{1}{2}$*d.* $\frac{1}{4}$

50. What is the price of 5 boxes of tea, each containing 37 lbs., at £4, 13*s.* $9\frac{3}{4}$*d.* for 17 lbs.? . . . . Ans. £51, 0*s.* $10\frac{3}{4}$*d.* $\frac{4}{17}$

51. How much will 41 yards 2 qrs. amount to at £22, 17*s.* 6*d.* for 25 English ells? . . . . . . Ans. £30, 7*s.* $6\frac{1}{2}$*d.* $\frac{22}{25}$

52. If the rent of 82 acres 3 roods 20 poles be £241, 17*s.* $2\frac{1}{2}$*d.*, how much will the rent of 10 acres 1 rood amount to at the same rate? . . . . Ans. £29, 18*s.* $3\frac{0}{4}$*d.* $\frac{544}{663}$

53. If I pay £10, 4*s.* 9*d.* of income-tax, being at the rate of 7*d.* in the pound, what is my annual income? . . Ans. £351.

54. If a man walk 7 miles in 2 hours 10 min., how many miles will he walk at the same rate in 4 hours? . Ans. $12\frac{12}{13}$ miles.

55. If 52 cwts. 13 lbs. of beef cost £158, 1*s.* $8\frac{1}{2}$*d.*, how much may be bought for £82, 7*s.* $2\frac{1}{2}$*d.*? . . Ans. 27 cwts. 17 lbs.

56. If 18 men can perform a piece of work in 28 days, how many men could do it in a fourth part of the time? . Ans. 72 men.

57. If a person can perform a journey in 6 days, riding 9 hours each day, how long will it take him to perform the same journey if he rides 12 hours a day? . . . . Ans. $4\frac{1}{2}$ days.

58. If 4 tons 6 cwts. of railway-bars cost £39, 17*s.* $5\frac{1}{4}$*d.*, how much must be paid for 1723 tons 1 qr. at the same rate?

Ans. £15,976, 13*s.* $8\frac{0}{4}$*d.* $\frac{53}{344}$

59. If 27 oxen are grazed in a field for 112 days, how many oxen could have been grazed equally well in the same field for 48 days? . . . . Ans. 63 oxen.

60. The rents of a parish amount to £1750, and a poor-rate is wanted of £98, 8*s.* 9*d.*; what must be the assessment per pound? . . . . Ans. 1*s.* $1\frac{1}{2}$*d.*

61. If the quartern-loaf cost $8\frac{1}{2}$*d.* when wheat is at 60*s.* per quarter, what ought to be its price when wheat is 49*s.* per quarter? . . . . Ans. $6\frac{3}{4}$*d.* $\frac{23}{30}$

62. A bankrupt's effects are valued at £983, 12*s.* 4*d.*, and his debts are £1728, 12*s.* $7\frac{1}{2}$*d.*, how much will his creditors receive per pound? . . . . Ans. 11*s.* $4\frac{1}{2}$*d.* $\frac{70224}{276581}$

63. If a clerk has a salary of £72, 18*s.* a year, commencing on the 1st of January, how much will he have to receive on leaving his situation on the 25th of September? Ans. £53, 10*s.* $6\frac{1}{4}$*d.* $\frac{187}{365}$

64. Sold 14 yds. 2 qrs. 1 nail for £10, 15*s.* 4*d.* from a piece of cloth which at first contained 28 yds. 1 qr.; what is the value of the remainder at the same rate? . . Ans. £10, 2*s.* $4\frac{1}{2}$*d.* $\frac{222}{233}$

65. If the carriage of 14 cwts. 0 qr. 23 lbs. for 65 miles comes to a certain sum of money, what weight may I have carried 37 miles for the same sum? . . . Ans. 24 cwts. 3 qrs. 23 lbs.

66. A vessel has provisions for 15 days, but being obliged by certain circumstances to continue at sea for 20 days, to what quantity must the daily ration of 20 lbs. be reduced to make the provisions last during that time? . . . . Ans. 15 lbs.

67. If the soldiers in a besieged garrison have provisions sufficient for 5 months at the rate of 20 oz. per man a day, how long will they be able to hold out when each man's allowance is reduced to 12 oz. a day? . . . . Ans. $8\frac{1}{3}$ months.

68. If a cistern of 230 gallons has a pipe which discharges 5 gallons in a minute, and another has a pipe which discharges 6 gallons in a minute, and if both cisterns are emptied in the same time, how many gallons does this last cistern contain?

Ans. 276 gals.

EXPLANATION OF THE RULE.—This rule depends on the principle that, from the nature of the question regarding the three given terms, there is always the same proportion, or ratio (as it is termed in mathematical language) between the two similar terms, as there is between the third term and the unknown quantity.

For example, in the question, 'If 10 yards of cloth cost £5, what will 20 yards cost?' it is obvious that there is a certain proportion between 10 yards and 20 yards—the one is the double of the other; and from the nature of the case, there must be a similar proportion between £5, the *price* of 10 yards, and the unknown sum, the price of 20 yards—the one will also be the double of the other.

To express the question distinctly, it is stated thus—as is the proportion of 10 yards to 20 yards, so is the proportion of £5 to the unknown sum. Omitting the words, the figures appear as follows, according to the rule of Simple Proportion:—

yds.		*yds.*		£	
10	:	20	::	5	:

The proportion between two numbers is ascertained by dividing the greater by the less—the proportion being expressed by the quotient: thus—if we divide 20 yards by 10 yards, the quotient is 2; that is, 20 is 2 times 10; or, *inversely*, 30 is the ½ of 20. In the present example, 20 yards being 2 times 10 yards, it follows that the *price* of 20 yards will be 2 times the price of 10 yards (£5); and therefore, if we multiply £5 by 2, we will obtain the unknown price required—namely, £10.

To express the process briefly: the principle of the rule consists in dividing the second term by the first, to find the proportion between them, and then multiplying the third term by the quotient or ratio, in order to find the unknown quantity.

In practice, instead of first dividing and then multiplying, as in No. 1, below, it is usually more convenient to multiply first, and then divide, as stated in the rule (see No. 2, below).

(1.)

```
yds.   yds.    £
 10 : )20  ::  5
       ---
        2
        5
       ---
       £10 Ans.
```

It will be observed that the result is the same in both cases.

(2.)

```
yds.   yds.    £
 10 :   20  :: 5
             5
          ----
        10)100
          ----
           £10 Ans.
```

Direct Proportion is where one number increases in proportion as another increases, or diminishes as another diminishes, as in example 1, page 57.

Inverse Proportion is where one number increases as another *diminishes*, or diminishes as another *increases*, as in example 2, page 57.

RATIO.—This is a mathematical term, as has been already stated, expressing the comparative magnitude of *two* numbers of the same kind; or how many times the one number is greater or smaller than the other: the ratio is ascertained by dividing the greater number by the less.

The ratio of a larger number to a smaller is expressed by the quotient on dividing the one by the other: thus—the ratio of 24 to 8 is 3; or as 3 to 1; meaning that 24 is 3 times 8.

The ratio of a smaller number to a larger, is expressed by the quotient *inversely*: thus—the ratio of 8 to 24 is ⅓; or as 1 to 3: meaning that 8 is one-third (⅓) of 24.

The two numbers of a ratio must always be of the same kind—as 10 pounds and 20 pounds; 50 miles and 100 miles. There cannot be a ratio between numbers of different kinds, such as 10 pounds and 100 miles.

The Rule of Proportion is founded on the *similarity* of the ratio of two given quantities, to the ratio of two other quantities.

COMPOUND PROPORTION.

Compound Proportion is the rule employed instead of Simple Proportion, when more than three terms (usually five) are given to find the unknown quantity; as, for example, if 12 persons earn £30 in 25 days, how much will 18 persons earn, at the same rate, in 56 days?

The principle of the rule is the same as in Simple Proportion.

RULES.

I. Rule for Stating.—1. Write down for the *third* term that number which is of the same kind as the answer required.

2. For the first and second terms, take two numbers of the same kind, and state them as in Simple Proportion, placing the greater and the less, first and second or the *reverse*, as the case requires.

3. Take other two terms of the same kind with each other, and state them in like manner, placing them directly under the preceding two; and so on with any others, till all the numbers are stated.*

II. Rule for Working.—1. Reduce the numbers of the first and second terms to the same denomination, and if the third term is compound, reduce it to a simple number, as in Simple Proportion.

2. Multiply the numbers of the first term together: then those of the second term; thus reducing the different numbers of these two terms into a single quantity for each.

3. Proceed with the three terms thus found, as in Simple Proportion.

Note.—The terms may be cancelled, when practicable, on the same principle as in Simple Proportion.

Example.—If I give 16 men £45 for 28 days' work, what must I give, at the same rate, to 20 men for 35 days' work?

```
16 men : 20 men :: £45
28 days  35 days
---      ---
128      700
32        45     £  s. d.
---     -----
448 ) 31500( 70  6  3 Ans.
```

Here money being the answer required, £45 is written for the third term: then 16 men and 20 men being terms of the same kind, 16 and 20 are written as the first and second, according to the rule in Simple Proportion: 28 days and 35 days being also of the same kind, 28 and 35 are placed respectively below 16 and 20. The 16 is then multiplied by 28, making 448 for the first term: the 20 is next multiplied by 35, making 700 for the second: the three terms, 448, 700, and 45, are then proceeded with as in Simple Proportion.

* The other method of stating Simple Proportion, page 59, may also be used in Compound Proportion.

Exercises.

1. If 12 persons consume 18 lbs. of beef in 4 days, how many pounds will 42 persons consume in 6 days? . . Ans. $94\frac{1}{2}$ lbs.

2. If 6 cows produce 28 gallons of milk in 2 days, how many gallons will 13 cows produce in seven days? Ans. $212\frac{1}{3}$ gallons.

3. If 17 boys wear among them 28 pair of shoes in a year, how many pair should 22 boys wear in a year and a half?

Ans. $54\frac{6}{17}$ pair.

4. A ship at sea had 112 persons on board, with provisions which gave an allowance of 1 lb. 6 oz. per day to each individual for 28 days; a starving ship's company, consisting of 49 persons, were picked up, increasing the number to 161; what quantity of provisions must all now be reduced to daily, in order to give each man a fair share during 26 days? . . Ans. $16\frac{144}{299}$ oz.

5. If 5 furnaces consume 30 tons of coal in 6 days, how many tons at the same rate of consumption should 3 furnaces consume in 11 days? . . . . Ans. 33 tons.

6. If the carriage of 2 tons 15 cwts. for 40 miles be £1, 8*s*., how much should be the carriage of 5 tons 11 cwts. for 72 miles? . . . . Ans. £5, 1*s*. $8\frac{1}{2}$*d*. $\frac{108}{275}$

7. If a person travel 120 miles in 4 days, by walking 9 hours a day, what time will be required for him to travel 386 miles, by walking 7 hours a day? . . . . . . Ans. $16\frac{18}{35}$ days.

8. If 75 men cut down 45 acres of corn in 4 days, how many acres will 108 men, working equally well, cut down in 25 days? . . . . Ans. 405 acres.

9. If 16 persons can be maintained for 60 days on £84, how much money would be required to support in similar circumstances 96 men for 365 days? . . . . . Ans. £3066.

10. If 18 compositors can set up 24 sheets in 8 days, how many sheets could 45 compositors set up in 14 days? Ans. 105 sheets.

11. If 25 horses consume 8 bushels of oats in 3 days, how many bushels would 42 horses consume in 15 days, at the same rate of living? . . . . Ans. $67\frac{1}{5}$ bushels.

12. If 3000 copies of a book of 11 sheets require 66 reams of paper, how much paper will be required for 5000 copies of a book of $12\frac{1}{2}$ sheets? . . . . Ans. 125 reams.

13. If a man can travel 360 miles in 12 days of 8 hours each, how many miles, at the same rate of walking, will he travel in 60 days of 6 hours each? . . . . . . Ans. 1350 miles.

14. If 18 men eat 15*s*. worth of bread in 3 days, when wheat is selling at 54*s*. per quarter, what value of bread will 54 men eat in 27 days, when wheat is selling at 50*s*. per quarter? Ans. £18, 15*s*.

15. A road-contractor engaged to finish $2\frac{1}{4}$ miles of road in 84 days; but after employing 60 men for 54 days, he found that they had only finished 880 yards; how many additional men must he engage, so that the work may be finished within the prescribed time? . . . . Ans. 372 men.

THE GREATEST COMMON MEASURE, OR DIVISOR.

THE GREATEST COMMON DIVISOR of two numbers, is the greatest number that will divide each of them, without a remainder: thus 5 is the greatest common divisor of 10 and 15.

TO FIND THE GREATEST COMMON DIVISOR OF TWO NUMBERS.

RULE.—Divide the greater by the less; then the divisor by the remainder if any; then the last divisor by the last remainder; and so on in the same way, till there is no remainder: the last *divisor* is the number sought.

This rule is useful in the reduction of Fractions, see page 68.

Example.—Find the greatest common divisor of 24 and 42.

```
24)42(1
   24
   18)24(1
      18
       6)18(3
         18
```

Here, after dividing 42 by 24, 24 by 18, and 18 by 6, the last divisor, 6, is found to be the number sought, as it divides 18, without a remainder.

Find the greatest common divisor of the following numbers:—

1. 252 and 348, . .	Ans. 12	3. 620 and 2108, . Ans. 124
2. 493 „ 899, . .	„ 29	4. 4081 „ 5141, . „ 53

THE LEAST COMMON MULTIPLE.

THE LEAST COMMON MULTIPLE of several numbers, is the *least* number that contains each of them a certain number of times exactly: thus—24 is the least common multiple of 4, 6, 8, and 12.

TO FIND THE LEAST COMMON MULTIPLE OF SEVERAL NUMBERS.

RULE.—Write the given numbers in a line, one after the other; cancel such of them as divide any of the others exactly; then divide as many of the rest as practicable, by some number* that divides them without a remainder, placing the quotients and any undivided numbers in the line below; again cancel, divide, &c., as before, carrying on the process till no numbers remain that have a common divisor: then multiply together the numbers used as divisors, and any undivided numbers in the last line; and the product is the least common multiple required.

This rule, like the preceding, is useful in the reduction of Fractions.

Example.—Find the least common multiple of 12, 4, 9, 30, 8.

$$\begin{array}{r|l} 2 & 12,\ \cancel{4},\ 9,\ 30,\ 8 \\ \hline 3 & 6,\ \ 9,\ 15,\ 4 \\ \hline & \cancel{2},\ \ 3,\ 5,\ 4 \end{array}$$

$2 \times 3 \times 3 \times 5 \times 4 = 360$ *Ans.*

Here, after cancelling and dividing according to the rule; the divisors employed and the numbers in the lowest line (which have no common divisor) are multiplied together for the answer.

Find the least common multiple of the following numbers:—

1. 2 4 5 8, . .	Ans. 40	3. 12 16 15 24, Ans. 240
2. 3 6 9 16, .	„ 144	4. 16 24 32 78, „ 1248

* Take whatever number will divide more of them than any other that could be taken.

VULGAR FRACTIONS.

A FRACTION means a part of a whole: the term is derived from a Latin word signifying *broken*.

Whole or unbroken numbers, as 1, 2, 3, &c., are termed integers; broken numbers as, $\frac{1}{2}$, a half; $\frac{1}{3}$, a third, &c., are termed fractions.

Fractions are of two kinds: *vulgar* fractions, from a Latin word signifying common; and *decimals*, from a word signifying ten.

VULGAR FRACTIONS are the common fractions of halves, thirds, fourths, and so on; the term is applied to all fractions when expressed by figures in this form—$\frac{2}{3}$, two-thirds; $\frac{5}{6}$, five-sixths, &c. They are called *vulgar* fractions, in distinction from *decimals*, a term applied to fractions of tenths, hundredths, &c. (see page 75), when expressed by figures in the same way as integers, except that a dot is placed before them, thus—·2, two-tenths.

If we suppose a loaf to be divided into two equal parts, each of the parts is a half, and forms a fraction of the whole; in figures, it is written as a Vulgar Fraction, thus—$\frac{1}{2}$.

Again, if the loaf is divided into four equal parts; each of these is called a *fourth*, or a *quarter*, and is written thus—$\frac{1}{4}$; two of them may be expressed as $\frac{2}{4}$, but as two-fourths are the same as one-half, they are written, $\frac{1}{2}$; three of them are written, $\frac{3}{4}$, expressing three-fourths. If the whole be divided into three equal parts, each part is called a *third*; if into five, each is called a *fifth*; if into six, a *sixth*; and so on, according to the number of parts into which the whole is divided; thus—$\frac{2}{3}$ means two-thirds of a whole; $\frac{3}{5}$, three-fifths; $\frac{5}{6}$, five-sixths.

To represent a Vulgar Fraction, therefore, two numbers are required, written the one above the other, with a short line between. The number *under* the line shews into how many parts the *whole* of the article, whatever it may be, is divided; and the number *above* the line, shews how many of these parts we mean to express.

The upper number is called the *numerator*, because it shews the *number* of the parts—as *three*-fourths, *six*-sevenths; the lower number is called the *denominator*, because it denominates the nature of the fraction—such as *thirds*, *eighths*, &c.

All the parts are together equal to the *whole*. Thus—two-halves, or three-thirds, or four-quarters, make each a *whole*.

When the upper figure of the fraction is *less* than the under, the fraction expresses *less* than a whole; when both numbers are the same, the fraction is equal to one whole; and when the upper number is greater than the under, it expresses more than one whole. Thus—$\frac{9}{13}$ is less than a whole, $\frac{4}{4}$ is equal to a whole, and $\frac{5}{4}$ is more than a whole.

It will now be understood that such quantities as $3\frac{5}{8}$ yards, or $2\frac{3}{4}$ pounds, mean 3 whole yards and five-eighth parts of a yard, and 2 whole pounds and three-fourth parts or 3 quarters of a pound.

An *improper fraction* is that of which the numerator is equal to or greater than the denominator—as $\frac{4}{4}$, $\frac{5}{4}$. The true way of expressing $\frac{4}{4}$ would be 1, and instead of $\frac{5}{4}$ it would be $1\frac{1}{4}$.

A *compound fraction* is a fraction of a fraction; as—$\frac{1}{5}$ of $\frac{3}{7}$.

A *mixed number* consists of an integer and a fraction—as $3\frac{6}{7}$.

Fractions may be *reduced*, *added*, *subtracted*, *multiplied*, or *divided*.

REDUCTION OF VULGAR FRACTIONS.

I. To reduce a Fraction, as $\frac{44}{76}$, to its lowest terms.

Rule.—Divide the numerator and denominator by any number that will divide both without a remainder; then divide the new fraction in a similar way; and continue the process till the fraction cannot be reduced any lower: this last fraction is the answer.

Or, when a divisor cannot be readily got, find the *greatest common divisor* (see page 66) of the numerator and denominator; then divide both by it, and the result will be the fraction in its lowest terms.

Note.—A fraction is not altered in value when both of its terms are multiplied or divided by the same number.

Example.—Reduce the fraction $\frac{44}{76}$ to its lowest terms.

$4\,)\frac{44}{76}(\,\frac{11}{19}$ *Ans.* Here we divide 44 and 76 by 4.

Exercises.—Reduce the following to their lowest terms:—

1. $\frac{8}{12}$	Ans.	$\frac{2}{3}$	4. $\frac{64}{128}$	Ans.	$\frac{1}{2}$	7. $\frac{875}{1000}$	Ans.	$\frac{7}{8}$
2. $\frac{54}{72}$	"	$\frac{3}{4}$	5. $\frac{120}{360}$	"	$\frac{1}{3}$	8. $\frac{3200}{5640}$	"	$\frac{80}{141}$
3. $\frac{74}{88}$	"	$\frac{37}{44}$	6. $\frac{164}{312}$	"	$\frac{41}{78}$	9. $\frac{785}{3250}$	"	$\frac{157}{650}$

II. To reduce a mixed number, as $2\frac{2}{5}$, to a fractional form.

Rule.—Multiply the integer by the denominator of the fraction, and include the numerator in the product; then write the denominator below it.

An integer is reduced to a fractional form by writing 1 below it; thus—4 is written as $\frac{4}{1}$.

Example.—Reduce $2\frac{2}{5}$ to a fractional form.

$$\begin{array}{l} 2\frac{2}{5} \\ \underline{5} \\ 12 = \frac{12}{5} \text{ Answer.} \end{array}$$

Exercises.—Reduce the following to a fractional form:—

1. $7\frac{4}{5}$	Ans.	$\frac{39}{5}$	3. 6	Ans.	$\frac{6}{1}$	5. $12\frac{11}{17}$	Ans.	$\frac{215}{17}$
2. $2\frac{3}{10}$	"	$\frac{23}{10}$	4. $9\frac{2}{3}$	"	$\frac{29}{3}$	6. $14\frac{5}{99}$	"	$\frac{1391}{99}$

III. To reduce an improper Fraction, as $\frac{17}{6}$, to a whole or mixed number.

Rule.—Divide the numerator by the denominator, and the quotient is either a whole or a mixed number, as the case may be.

Example—Reduce $\frac{17}{6}$ to a mixed number.

$$\begin{array}{l} 6\,)\underline{17} \\ \quad 2\frac{5}{6} \text{ Answer.} \end{array}$$

Exercises.—Reduce the following to whole or mixed numbers:—

1. $\frac{12}{6}$	Ans.	2	3. $\frac{6}{4}$	Ans.	$1\frac{1}{2}$	5. $\frac{21}{7}$,	Ans.	3
2. $\frac{10}{3}$	"	$3\frac{1}{3}$	4. $\frac{33}{2}$	"	$16\frac{1}{2}$	6. $\frac{39}{4}$,	"	$9\frac{3}{4}$

IV. To reduce a compound to a simple Fraction.

Rule.—Multiply all the numerators together for the numerator, and all the denominators together for the denominator; the resulting fraction may then be reduced to its lowest terms.

Example.—Reduce $\frac{5}{6}$ of $\frac{3}{12}$ to a simple fraction.

$$\frac{5}{6} \times \frac{3}{12} \doteq \frac{15}{72} = \frac{5}{24} \quad \textit{Answer.}$$

Exercises.—Reduce the following to simple fractions :—

1. $\frac{2}{3}$ of $\frac{1}{2}$	Ans. $\frac{1}{3}$	3. $\frac{9}{10}$ of $\frac{6}{12}$	Ans. $\frac{9}{20}$	5. $\frac{2}{3}$ of $\frac{3}{5}$	Ans. $\frac{2}{5}$
2. $\frac{3}{7}$ 〃 $\frac{2}{8}$	〃 $\frac{3}{28}$	4. $\frac{7}{8}$ 〃 $\frac{4}{11}$	〃 $\frac{7}{22}$	6. $\frac{3}{7}$ 〃 $\frac{5}{9}$	〃 $\frac{5}{21}$

V. To convert Fractions having different denominators, to other equivalent Fractions having a common denominator.

Rule 1. Find the least common multiple (see page 66) of all the denominators, and place it as the common denominator; then multiply this common denominator by the upper figure of each fraction, and divide the product by the under figure of each, for the respective numerators: the resulting fractions will be in the lowest *common* terms, if the given fractions are stated in their lowest terms.

Rule 2. Another method is to multiply all the denominators together for a common denominator; then, as before, multiply this common denominator by the upper figure, and divide it by the under figure of each fraction, for the respective numerators: the resulting fractions may then be reduced to their lowest common terms.

Example.—Convert $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{6}$, to other fractions having a common denominator.

First Method.

30	30	30
2	3	5
3) 60	5) 90	6) 150
20	18	25
Ans. $\frac{20}{30}$	$\frac{18}{30}$	$\frac{25}{30}$

Here the least common multiple of the denominators is found to be 30, and is multiplied, divided, &c., according to the rule.

Second Method.

90	90	90
2	3	5
3) 180	5) 270	6) 450
60	54	75
Ans. $\frac{60}{90} = \frac{20}{30}$	$\frac{54}{90} = \frac{18}{30}$	$\frac{75}{90} = \frac{25}{30}$

Here all the denominators are multiplied together for a common denominator; $3 \times 5 \times 6 = 90$: and this is multiplied, divided, &c., as before: the resulting fractions are reduced to their lowest common terms.

Exercises.—Convert to fractions having a common denominator:

1. $\frac{3}{6}$ $\frac{5}{8}$	Ans. $\frac{4}{8}$ $\frac{5}{8}$	4. $\frac{3}{10}$ $\frac{4}{25}$ $\frac{8}{15}$	Ans. $\frac{45}{150}$ $\frac{24}{150}$ $\frac{80}{150}$
2. $\frac{2}{3}$ $\frac{6}{9}$ $\frac{7}{36}$	〃 $\frac{24}{36}$ $\frac{24}{36}$ $\frac{7}{36}$	5. $\frac{2}{7}$ $\frac{3}{8}$ $\frac{10}{21}$	〃 $\frac{48}{168}$ $\frac{63}{168}$ $\frac{80}{168}$
3. $\frac{2}{3}$ $\frac{2}{7}$ $\frac{1}{2}$	〃 $\frac{28}{42}$ $\frac{12}{42}$ $\frac{21}{42}$	6. $\frac{7}{36}$ $\frac{13}{48}$ $\frac{17}{32}$	〃 $\frac{56}{288}$ $\frac{78}{288}$ $\frac{153}{288}$

VI. To convert a Fraction of one Denomination into the Fraction of another, without altering its Value.

Rule.—Ascertain how many of the smaller denomination make *one* of the greater: if the conversion is from a higher to a lower denomination, multiply the *numerator* of the fraction by that number; if from a lower to a higher, multiply the *denominator*.

Example 1.—Convert $\frac{2}{630}$ of a pound to the fraction of a penny.

$$\frac{2}{630} \times 240 = \frac{480}{630} = \frac{16}{21} \text{ of } 1d. \textit{ Ans.}$$

Here, as the change is from a higher to a lower, we multiply the *numerator* by 240, the number of pence in a pound.

Example 2.—Convert $\frac{2}{3}$ of a penny to the fraction of a shilling.

$$\frac{2}{3 \times 12} = \frac{2}{36} = \frac{1}{18} \text{ of } 1s. \textit{ Ans.}$$

Here, as the change is from a lower to a higher, we multiply the *denominator* by 12, the number of pence in a shilling.

Exercises.

1.	Convert	$\frac{1}{4}$	of a	farthing	to the	fraction of	$1d.$,	Ans.	$\frac{1}{16}$	of	$1d.$
2.	"	$\frac{2}{5}$	"	penny	"	"	$1s.$	"	$\frac{1}{30}$	"	$1s.$
3.	"	$\frac{3}{7}$	"	shilling	"	"	£1,	"	$\frac{3}{140}$	"	£1.
4.	"	$\frac{2}{9}$	"	shilling	"	"	$\frac{1}{4}d.$	"	$\frac{32}{3}$	"	$\frac{1}{4}d.$
5.	"	$\frac{3}{5}$	"	pound	"	"	$\frac{1}{4}d.$	"	$\frac{576}{1}$	"	$\frac{1}{4}d.$
6.	"	$\frac{1}{420}$	"	pound	"	"	$1d.$	"	$\frac{4}{7}$	"	$1d.$
7.	"	$\frac{3}{480}$	"	shilling	"	"	$\frac{1}{4}d.$	"	$\frac{3}{10}$	"	$\frac{1}{4}d.$
8.	"	$\frac{2}{340}$	"	ton	"	"	1 qr.	"	$\frac{8}{17}$	"	1 qr.

VII. To find the Value of a Fraction of a given Denomination.

Rule.—Reckon the upper figure of the fraction as so many of the given denomination, and then divide it by the under figure, as in Compound Division.

Example.—What is the value of $\frac{2}{3}$ of a pound?

$$\begin{array}{r} £2 \\ 20 \\ 3\,)\overline{40} \\ \hline 13s.\ 4d. \end{array} \textit{ Ans.}$$

Here the 2 is reckoned as £2, and divided by 3, as in Compound Division.

Exercises.—What is the value of—

1.	$\frac{2}{3}$ of a shilling,	Ans. 8*d.*	4.	$\frac{4}{7}$ of a shilling,	Ans. $6\frac{3}{4}d.\ \frac{3}{7}$
2.	$\frac{5}{8}$ " pound,	" 12*s.* 6*d.*	5.	$\frac{3}{11}$ " pound,	" $5s.\ 5\frac{1}{4}d.\ \frac{9}{11}$
3.	$\frac{5}{24}$ " pound,	" 4*s.* 2*d.*	6.	$\frac{9}{12}$ " ton,	" 15 cwts.

VIII. To convert a given sum, as 14*s.* 6*d.*, to a Fraction of another denomination.

Rule.—Convert the given sum into its lowest denomination, and place it as the numerator; then convert *one* of the proposed denomination into the same denomination as the other, and place it as the denominator: the resulting fraction may then be reduced to its lowest terms.

Example.—Convert 3*s.* 3*d.* to the fraction of a pound.

$$\begin{array}{lll} 3s.\ 3d., & \text{reduced to pence, its lowest denomination} = & 39 \\ £1, & \text{reduced to pence} \quad . \quad . \quad . \quad . \quad . \quad = & 240 \end{array} \qquad \frac{39}{240} = \frac{13}{80} \text{ of } £1.$$

Exercises.

1. Convert 6*s.* 9*d.* to the fraction of a pound, . . Ans. £$\frac{27}{80}$
2. " 14*s.* 7½*d.* " " . " £$\frac{117}{160}$
3. " 16*s.* 8*d.* " " . . " £$\frac{5}{6}$
4. " 1 qr. 12 lbs. to the fraction of a cwt., Ans. $\frac{5}{14}$ cwt.

ADDITION OF VULGAR FRACTIONS.

Rule.—Convert the fractions, if they have different denominators, into others having a common denominator; then add together all the numerators, and under the sum write the common denominator: the resulting fraction may then be reduced to its lowest terms.

If the answer is an improper fraction, reduce it to a whole or mixed number.

Note.—When mixed numbers are given, add the fractions first, and then the integers: when fractions are compound, before adding, convert them into simple fractions; and if of different denominations, convert them into the same denomination.

Example.—Add together $\frac{4}{5}$, $\frac{2}{3}$, $\frac{3}{10}$.

$$\frac{4}{5} = \frac{120}{150}$$
$$\frac{2}{3} = \frac{100}{150}$$
$$\frac{3}{10} = \frac{45}{150}$$
$$\frac{120 + 100 + 45 = 265}{\text{Common denominator } 150} = \frac{53}{30} = 1\frac{23}{30} \textit{ Ans.}$$

These are first converted to fractions having a common denominator, by Reduction, Rule V.,* page 69; the numerators are then added together, and the common denominator written below their amount: the answer being an improper fraction, is converted into the mixed number $1\frac{23}{30}$.

* The second method under Rule V., is used in this example.

Exercises.—Add together—

1. $\frac{2}{7}$ $\frac{3}{4}$ Ans. $1\frac{1}{28}$
2. $\frac{1}{9}$ $\frac{3}{8}$ $\frac{4}{6}$ " $1\frac{11}{72}$
3. $\frac{1}{2}$ $\frac{3}{12}$ $\frac{4}{7}$ " $1\frac{9}{28}$
4. $\frac{4}{9}$ $\frac{6}{7}$ $\frac{10}{11}$ Ans. $2\frac{146}{693}$
5. $\frac{2}{3}$ of $\frac{1}{7}$ $\frac{3}{7}$ of $\frac{5}{12}$ of $4\frac{1}{2}$ " $\frac{151}{168}$
6. $12\frac{1}{2}$ $24\frac{3}{4}$ $17\frac{5}{8}$ " $54\frac{7}{8}$

SUBTRACTION OF VULGAR FRACTIONS.

Rule.—Convert the fractions, when of different denominators, into others having a common denominator; then subtract the numerator of the one from that of the other, and under the difference write the common denominator: the resulting fraction may then be reduced to its lowest terms.

Note.—When mixed numbers are given, before subtracting, convert them into improper fractions: also, convert compound into simple fractions, and if of different denominations, convert them into the same denomination.

Example.—From $\frac{6}{7}$ take $\frac{4}{5}$.

$$\frac{6}{7} = \frac{30}{35}$$
$$\frac{4}{5} = \frac{28}{35}$$
$$\frac{30 - 28}{\text{Common denominator } 35} = \frac{2}{35} \textit{ Ans.}$$

These are first converted to fractions having a common denominator; the numerator of the one is then subtracted from that of the other, and the common denominator written below the difference.

Exercises.—What is the difference between—

1.	$\frac{2}{3}$ and $\frac{3}{5}$	Ans. $\frac{1}{15}$	4. $\frac{1}{2}$ and $\frac{9}{11}$	Ans. $\frac{7}{22}$
2.	$\frac{4}{7}$ 〃 $\frac{6}{8}$	〃 $\frac{5}{28}$	5. $\frac{2}{3}$ of $\frac{9}{10}$ 〃 $\frac{1}{2}$ of $\frac{3}{7}$	〃 $\frac{27}{70}$
3.	$\frac{5}{9}$ 〃 $\frac{7}{10}$	〃 $\frac{13}{90}$	6. 49 〃 $23\frac{5}{8}$	〃 $25\frac{3}{8}$

MULTIPLICATION OF VULGAR FRACTIONS.

Rule.—Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator: the resulting fraction may then be reduced to its lowest terms.

Note.—Before multiplying, convert any mixed number into improper fractions, and compound into simple fractions: also, in order to shorten the multiplication, the fractions may be reduced to their lowest terms.

A fraction is multiplied by an integer by multiplying its upper figure only: thus $\frac{3}{4}$ multiplied by 6 are $\frac{18}{4} = 4\frac{1}{2}$.

The result of multiplying by a fraction is to *lessen* the multiplicand. This will be obvious if we consider that to multiply by 1 does not increase a number, but leaves it the same as before, and consequently to multiply by *less* than 1—that is, by a fraction—must diminish it.

Example.—Multiply $\frac{7}{8}$ by $\frac{4}{9}$.

$$\frac{7}{8} \times \frac{4}{9} = \frac{28}{72} = \frac{7}{18} \textit{ Answer.}$$

Exercises.—Multiply the following—

1.	$\frac{4}{5}$ by $\frac{6}{7}$	Ans. $\frac{24}{35}$	4. $5\frac{1}{6}$ by $\frac{12}{15}$	Ans. $4\frac{2}{15}$
2.	$\frac{9}{10}$ 〃 $\frac{7}{12}$	〃 $\frac{21}{40}$	5. $\frac{3}{4}$ of $\frac{3}{5}$ 〃 $12\frac{1}{2}$	〃 $5\frac{5}{8}$
3.	$4\frac{2}{3}$ 〃 $\frac{8}{15}$	〃 $2\frac{22}{45}$	6. $79\frac{3}{16}$ 〃 $9\frac{1}{3}$	〃 $739\frac{1}{12}$

DIVISION OF VULGAR FRACTIONS.

Rule.—Invert the given divisor—thus, if it is $\frac{3}{7}$, write it as $\frac{7}{3}$; then multiply the two fractions together, and the resulting fraction is the quotient: it may then be reduced to its lowest terms.

Note.—Before proceeding, convert any mixed numbers into improper fractions, and compound into simple fractions.

A fraction is divided by an integer by dividing its upper figure only; thus $\frac{8}{9}$ divided by 4 are $\frac{2}{9}$.

The result of dividing by a fraction is to *increase* a number, on the same principle that multiplying lessens it. See Multiplication, page 72.

Example.—Divide $\frac{3}{5}$ by $\frac{6}{7}$.

$$\frac{3}{5} \times \frac{7}{6} = \frac{21}{30} = \frac{7}{10} \textit{ Ans.}$$

Here the divisor $\frac{6}{7}$ is written as $\frac{7}{6}$; the two fractions are then multiplied together, and the resulting fraction reduced to its lowest terms.

Exercises.—Divide the following—

1.	$\frac{6}{7}$ by $\frac{4}{5}$	Ans. $1\frac{1}{14}$	4.	8 by $\frac{2}{5}$	Ans. 20
2.	$\frac{9}{10}$ 〃 $\frac{3}{4}$	〃 $1\frac{1}{5}$	5.	$\frac{7}{9}$ of $4\frac{1}{2}$ 〃 $\frac{2}{3}$ of $\frac{5}{6}$	〃 $6\frac{3}{10}$
3.	$7\frac{1}{6}$ 〃 $\frac{2}{3}$	〃 $10\frac{3}{4}$	6.	$46\frac{1}{2}$ 〃 $12\frac{3}{4}$	〃 $3\frac{11}{17}$

Miscellaneous Exercises in Vulgar Fractions.

1. What is the sum of $\frac{2}{3}$ and $\frac{3}{4}$? . . . . . Ans. $1\frac{5}{12}$
2. If $\frac{1}{3}$, $\frac{2}{7}$, and $\frac{5}{6}$ be added together, what is the amount? 〃 $1\frac{19}{42}$
3. If we take $\frac{1}{9}$ from $\frac{2}{7}$, what remains? . . . . 〃 $\frac{11}{63}$
4. What is the difference between $\frac{4}{5}$ and $\frac{3}{8}$? . 〃 $\frac{17}{40}$
5. Whether is $\frac{3}{7}$ or $\frac{5}{8}$ the greater number, and what is the excess? . . . . . . Ans. $\frac{5}{8}$ is greater by $\frac{11}{56}$
6. How many sevenths are there in three and two-sevenths? Ans. 23 sevenths.
7. Multiply $\frac{5}{6}$ by $\frac{7}{8}$, ... Ans. $\frac{35}{48}$
8. 〃 $\frac{3}{7}$ 〃 $\frac{4}{9}$, ... 〃 $\frac{4}{21}$
9. 〃 $\frac{15}{17}$ 〃 $\frac{2}{3}$, ... 〃 $\frac{10}{17}$
10. Divide $\frac{3}{8}$ 〃 $\frac{4}{5}$, ... 〃 $\frac{15}{32}$
11. 〃 $\frac{2}{11}$ 〃 $\frac{5}{12}$, ... 〃 $\frac{24}{55}$
12. 〃 $\frac{1}{9}$ 〃 $\frac{3}{5}$, ... 〃 $\frac{5}{27}$
13. 〃 three-elevenths by eleven-twelfths, . . . 〃 $\frac{36}{121}$
14. What is the half of a half? . . . . . 〃 $\frac{1}{4}$
15. How much is two-thirds of two-thirds? . . . 〃 $\frac{4}{9}$
16. Of what number is $\frac{4}{9}$ the double? . . . 〃 $\frac{2}{9}$

17. What number multiplied by $\frac{2}{7}$ gives $\frac{2}{9}$? . . . Ans. $\frac{7}{9}$

18. What number is that, $\frac{1}{3}$ of which is five-elevenths? 〃 $1\frac{4}{11}$

19. What is the seventh part of the half of three-eighths? 〃 $\frac{3}{112}$

20. If two-thirds of 1 lb. cost $5\frac{1}{2}d.$, what will three pounds and a quarter cost? . . . . Ans. 2*s*. $2\frac{3}{4}d.$ $\frac{1}{4}$

21. What is the half of a third part of £1 sterling? Ans. 3*s*. 4*d*.

22. What part of £1 is 17*s*. 6*d*.? . . . . . Ans. $\frac{7}{8}$

23. If three-eighths of a ship be worth £15,000, what is the value of the whole vessel? . . . . . . Ans. £40,000

24. If a whole ship is valued at £50,000, what are three sixty-fourths of it worth? . . . . . . Ans. £2343, 15*s*.

25. If one man can do a piece of work in three days, how much of it can he do in a day? . . . . Ans. $\frac{1}{3}$

26. If five men can execute a job in one day, how much can each of them perform in the same time? . . . . Ans. $\frac{1}{5}$

27. If one man require three days to complete an undertaking, and another five, how much will they do in one day, if they work together at the same rate? . . . . Ans. $\frac{8}{15}$

28. If a man can perform three-eighths of a piece of work in one day, in what time will he finish it? . . . Ans. $2\frac{2}{3}$ days.

29. A person pays away two-thirds of his money, and gets back two shillings; what had he at first, if he has now the half of his original sum? . . . . Ans. 12*s*.

30. A man spends half of his money at one shop, and the third of the remainder at another; he finds that he has six shillings left; what was the sum the man originally had? . Ans. 18*s*.

31. If we cut an apple into five parts, and one of these parts into seven others, what part of the whole apple is one of the smaller parts? . . . . Ans. The thirty-fifth part.

32. Whether is three-fourths of 3 or four-thirds of 4 the greater number? . . . . Ans. $\frac{4}{3}$ds of 4, by $3\frac{1}{12}$

33. If a man give away three-sevenths of £14, how much money has he left? . . . . Ans. £8

34. If a man who has twenty-five guineas, give away two-fifths of it to one man, two-fifths of the remainder to a second, and the rest to a third, how much money does each of them receive?

Ans. The first, £10, 10*s*.; Second, £6, 6*s*.; Third, £9, 9*s*.

35. If one cock empty a cistern in five, and another in four hours, in what time will they empty it if used together? Ans. $2\frac{2}{9}$ hours.

36. If a greyhound's leap is one-fourth longer than a hare's, in what time will it overtake the hare that is fifty leaps before it, and that they both take one leap in a second? Ans. 3 min. 20 sec.

37. Two men set out at the same time, the one from London to York, and the other from York to London; the one travels at the rate of $5\frac{1}{2}$ miles an hour, and the other $6\frac{1}{4}$; in how many hours will they meet, the distance between the two places being 197 miles? . . . . Ans. $16\frac{36}{47}$ hours.

DECIMAL FRACTIONS.

DECIMAL FRACTIONS are those which express tenths, or combinations of tenths, and are usually indicated by figures in the same way as integers, except that a *point* is placed before them for distinction: thus—·3, three-tenths; ·47, forty-seven hundredths. The term is derived from the Latin word *decem*, signifying ten.

In *decimal* fractions, instead of the numerator and denominator both being expressed as in *Vulgar* Fractions, the numerator only is written down (hence the fraction is written in the same way as integers), and the denominator is always understood to be either 10, 100, 1000, or some other combination of tenths, according to the number of figures in the numerator. If the numerator consists of 1 figure, the understood denominator is 10; if of 2 figures, it is 100, and so on; the denominator being always 1, with as many nothings annexed as there are figures in the numerator: for example—$\frac{9}{10}$ are written decimally ·9; or $\frac{99}{100}$ as ·99. An integer with a fraction, as $3\frac{9}{10}$, is written thus—3·9; the *point* [·] marking the separation between them.

As in whole numbers, a figure *increases* in value ten times by every removal to the *left* of the units' place, so in decimals, a figure *decreases* in value ten times by every removal to the *right* of the units' place: the first figure to the *right* of the point, or, as it is termed, the first *place* of decimals, expresses so many *tenths;* the second figure to the right, so many *hundredths;* the third figure, so many *thousandths;* and so on, according to the following table :—

·1 . . . =	$\frac{1}{10}$	or 1-tenth.		·1 - 10th.
·02 . . =	$\frac{2}{100}$	" 2-hundredths.		·12 - 100ths.
·003 . . =	$\frac{3}{1000}$	" 3-thousandths.		·123 - 1000ths.
·0004 . =	$\frac{4}{10,000}$	" 4-ten-thousandths.		·1234 - 10,000ths.
·00005 . =	$\frac{5}{100,000}$	" 5-hundred-thousandths.		·12345 - 100,000ths.
·000006 =	$\frac{6}{1,000,000}$	" 6-millionths.		·123456 - 1,000,000ths.

Decimals are read from left to right, as in whole numbers, beginning at the first figure after the point, and naming them according to the number of parts expressed by the figures; thus—5·375, is read five, and three hundred and seventy-five *thousandths*.

The *annexing* of nothings to decimals does not alter or increase their value: thus ·5 and ·500 express the same value, because ·5 bears the same proportion to 10, that ·500 does to 1000, each being equal to one-half ($\frac{1}{2}$).

The *prefixing* of nothings to decimals decreases their value tenfold for every nothing prefixed: thus ·1 ($\frac{1}{10}$) becomes ·01 ($\frac{1}{100}$) by prefixing *one* nothing.

The calculations in Decimals are performed in the same way as in integers, and hence their great convenience and utility.

Decimals are useful in astronomical, chemical, and other scientific calculations, which require much nicety and minuteness.

Decimals may be *reduced*, *added*, *subtracted*, *multiplied*, or *divided*.

REDUCTION OF DECIMALS.

I. To convert a Vulgar Fraction into a Decimal.

Rule.—Annex as many nothings to the numerator as will make it greater than the denominator, and then divide it by the latter; continuing to annex nothings and to divide till there is no remainder, or till the same figures come to be repeated in the quotient.

The answer must contain as many decimals as there have been nothings annexed to the numerator; if there are not as many after dividing, *prefix* the requisite number to the quotient.

Note.—It is only some Vulgar Fractions that admit of being exactly expressed by decimals, as in Example 1, where, on dividing, the fraction terminates exactly in the decimal; which is hence called a *terminate* decimal. There are other fractions, as in Examples 2 and 3, which cannot be exactly expressed by decimals, as they do not terminate exactly; however, by carrying the division to several places of decimals, as 8333 &c., the difference between the decimal and the exact fraction becomes too trifling to be appreciable: such decimals are called *interminate.*

Example 1.—Convert the Vulgar Fraction $\frac{3}{8}$ into a decimal.

$$8)\underline{3000}$$
$$\cdot 375 \text{ Ans.}$$

Here 3 nothings require to be annexed before the division comes to a close; the answer, therefore, consists of 3 decimals.

Example 2.—Convert $\frac{5}{6}$ into a decimal.

$$6)\underline{500}$$
$$\cdot 83 \text{ \&c., Ans.}$$

This is an example of an *interminate* decimal: after annexing two nothings, and dividing by 6, the answer is ·83 and 2 over: by annexing more nothings, the division might be carried to an unlimited extent, as the decimals would always be 3 repeated continually, with a remainder of 2 over, however many nothings might be annexed.

Example 3.—Convert $\frac{7}{22}$ to a decimal.

$$22\,)\underline{700000}$$
$$31818 \text{ \&c., Ans.}$$

Here the division may also be carried to an unlimited extent, by annexing more nothings: the rest of the decimals would always be 18 repeated.

Exercises.—Reduce the following to decimals :—

1.	$\frac{1}{5}$	Ans. ·2	4.	$\frac{3}{4}$	Ans. ·75	7.	$\frac{7}{625}$	Ans. ·0112
2.	$\frac{1}{2}$	" ·5	5.	$\frac{1}{3}$	" $\cdot\dot{3}$	8.	$\frac{4}{13}$	" $\cdot\dot{3}0769\dot{2}$
3.	$\frac{1}{4}$	" ·25	6.	$\frac{12}{25}$	" ·48	9.	$\frac{3}{14}$	" $\cdot 2\dot{1}4285\dot{7}$

The term, Recurring Decimal, is applied to those decimals in which the same figure or figures are continually repeated: they are called *Repeating*, or *Circulating*, according to the number of figures repeated.

A *repeating* decimal is where the *same* figure is repeated, and is indicated by a dot placed over the recurring figure; thus, ·833 &c., is written as $\cdot 8\dot{3}$.

A *circulating* decimal is where two or more figures are repeated, and is indicated by a dot over the first and last recurring figures; thus ·31818 &c., is written as $\cdot 3\dot{1}\dot{8}$, and ·73925925 &c., as $\cdot 73\dot{9}2\dot{5}$.

When the decimal consists entirely of recurring figures, as $\cdot\dot{3}$, it is termed a *pure* recurring decimal: when it consists partly of recurring, and partly of *non*-recurring figures, as $\cdot 8\dot{3}$, it is termed a *mixed* recurring decimal.

II. To convert a *terminate* Decimal into a Vulgar Fraction.

Rule.—Write the given decimal as the numerator, and for the denominator, write 1 and as many nothings as there are figures in the decimal: then reduce the fraction thus obtained to its lowest terms.

Example.—Reduce ·75 to a vulgar fraction.

$$\frac{75}{100} = \frac{3}{4} \textit{ Answer.}$$

Exercises.—Reduce the following to vulgar fractions—

1.	·5	Ans. $\frac{1}{2}$	3.	·25	Ans. $\frac{1}{4}$
2.	·04	" $\frac{1}{25}$	4.	·78	" $\frac{39}{50}$

Note 1.—A *pure* recurring decimal is converted into a vulgar fraction by writing as many *nines* for the denominator as there are figures in the decimal, thus $\cdot\dot{7}$ is written $\frac{7}{9}$; $\cdot\dot{8}\dot{1}$ as $\frac{81}{99} = \frac{9}{11}$.

Note 2.—A *mixed* recurring decimal is converted into a vulgar fraction as follows:—Subtract the *non*-recurring figures from the decimal, and write the remainder for the numerator: then for the denominator write as many *nines* as there are *recurring* figures, and annex to them as many nothings as there are *non*-recurring figures: the resulting fraction may then be reduced to its lowest terms.

Example.—Convert $72\dot{3}\dot{6}$ to a vulgar fraction.

$$72\dot{3}\dot{6} \text{ less } 72 = \frac{7164}{9900} = \frac{199}{275}$$

Here we deduct the non-recurring figures, 72, from the decimal, leaving 7164 for the numerator, and then write two nines and two nothings for the denominator: the resulting fraction is then reduced to its lowest terms.

III. To convert a given sum, as 2*s.* 6*d.*, to the Decimal of a higher Denomination.

Rule.—Convert the given sum, when compound, to its lowest denomination; convert also *one* of the specified higher denomination to the same denomination as the other; annex as many nothings to the former as will make it greater than the latter; then divide the one by the other, continuing to annex nothings and to divide till there is no remainder, or as far as the division is wished to be carried. There must be as many decimals in the answer as there have been ciphers annexed.

Example.—Reduce 2*s.* 6*d.* to the decimal of a pound.

```
     2s. 6d.
     12
240)30000
    £·125 Ans.
```

Here 2*s.* 6*d.* is reduced to its lowest denomination, pence = 30, and one of the specified higher denomination, pounds, is also reduced to pence = 240; nothings are then annexed to 30, and the dividend divided by 240. As three nothings have been annexed, there are three decimals in the answer.

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Exercises.

1. Convert 12*s.* 6*d.*, 5*s.* 4*d.*, and 6*s.* 3¼*d.* to the decimal of £1, Ans. £·625, £$\cdot 2\dot{6}$, £$\cdot 31458\dot{3}$
2. " 17*s.* 5¾*d.*, 18*s.* 7½*d.*, and 13*s.* 6*d.* to the decimal of £1, Ans. £$\cdot 873958\dot{3}$, £·93125, £·675
3. " 3*s.* 10¼*d.*, 19*s.* 9*d.*, and 16*s.* 8*d.* to the decimal of £1, Ans. £$\cdot 192708\dot{3}$, £·9875, £$\cdot 8\dot{3}$
4. " 4*d.*, 6*d.*, and 8*d.* to the decimal of £1, Ans. £$\cdot 01\dot{6}$, £·025, £$\cdot 0\dot{3}$
5. " 2 qrs. 17 lbs. to the decimal of a cwt., Ans. $\cdot 651\dot{7}85714\dot{2}$ cwt.
6. " 3 cwts. 3 qrs. 8 lbs. to the decimal of a ton, Ans. $\cdot 1910\dot{7}1428\dot{5}$ ton.

NOTE.—To CONVERT SHILLINGS, PENCE, AND FARTHINGS into the decimals of a pound, the following is a convenient rule in practice:

Reckon *half* the number of the shillings as the first decimal; thus, consider 12*s.* as ·6: if the number of shillings is odd, as 13*s*, carry the odd 1*s.* to the pence, convert the pence, also the odd 1*s.* into farthings, and include any farthings that are in the given sum; then reckon the farthings as the second and third decimals, adding 1 for every 24; thus, if the farthings come to 54, reckon the decimal as 56 (for twice 24). This rule gives the answer nearly correct; it will never be more than ½ too much or too little.

Example.—Convert 13*s.* 6½*d.* into the decimals of £1, . Ans. £·677

Here the half of 13*s.* is 6*s.* and 1 over: the 6 is placed as the first decimal; then the odd 1*s.* and the 6½*d.* = 1*s.* 6½*d.* are 74 farthings, and adding 3 farthings (for thrice 24), make 77 farthings, which are placed as the second and third decimals.

Exercises.

1.	7*s.* 6*d.*	=	·375	3.	4*s.* 6*d.*	=	·225
2.	3*s.* 4*d.*	=	·166	4.	18*s.* 4*d.*	=	·916

IV. TO FIND THE VALUE OF A DECIMAL OF A GIVEN DENOMINATION.

RULE.—Reckon the decimal as so many of the given denomination; then divide it, as in Compound Division, by 10, if it consist of one figure; by 100, if it consist of two figures; and so on; using always as many nothings as there are figures in the decimal.

Example.—Find the value of £·375.

```
£·375
   20
-----
7,500
   12
-----
6,000
Ans. 7s. 6d.
```

Here, £·375 is reckoned as £375, and as there are three figures in the decimal, we divide by 1000, according to Compound Division, Rule III., page 52; there being no pounds in the answer, we reduce £375 to shillings, and point off 3 figures, leaving 7*s.*; then reducing the figures pointed off to pence, we again point off 3 figures, leaving 6*d.*

Exercises.

1.	Find the value of	·75*s*., ...	Ans. 9*d.*
2.	" "	£·85, ...	" 17*s.* 0*d.*
3.	" "	£·450, ...	" 9*s.* 0*d.*
4.	" "	£·675, ...	" 18*s.* 6*d.*
5.	" "	·425 tons, .	Ans. 8 cwts. 2 qrs.
6.	" "	£·6375; £·78125,	" 12*s.* 9*d.*; 15*s.* 7½*d.*
7.	" "	£1·928125; £·15625,	" £1, 18*s.* 6¾*d.*; 3*s.* 1½*d.*
8.	" "	1·475 tons, . .	" 1 ton 9 cwts. 2 qrs.

NOTE.—THE DECIMALS OF A POUND may in practice be conveniently valued by the following Rule, three decimal places being taken.

Reckon the double of the first decimal as so many shillings, and the second and third decimals as so many farthings—less 1 farthing for every 25: thus—to find the value of £·364, reckon 3 as 6*s*, and 64 as 64 farthings, less 2 (for twice 25), making 62 farthings or 1*s.* 3½*d.*, and the answer is 7*s.* 3½*d.* This rule will give the answer nearly correct: it will never be more than ¼*d.* too much or too little.

Exercises.

1.	£·182	=	3*s.* 7¾*d.*	3.	£·825	=	16*s.* 6*d.*
2.	£·375	=	7*s.* 6*d.*	4.	£·924	=	18*s.* 5¾*d.*

ADDITION OF DECIMALS.

RULE.—Write down the numbers in such a way that the *points* shall be directly under each other; thus having units under units, tens under tens, &c., in integers; and in decimals, tenths under tenths, hundredths under hundredths, and so on; then proceed as in Simple Addition: the decimal point in the answer is placed directly below the other points.

Example.—234·678
39·76
456·2307
730·6687 *Answer.*

Exercises.

1.	Add	637·4,	295·76,	4586·314,	. . Ans.	5519·474
2.	"	9863·5,	275·146,	3912·78650,	. "	14051·4325
3.	"	7·9654,	10·12450,	46·754361,	. . "	64·844261
4.	"	298·65,	475·672,	54·89008,	. "	829·21208
5.	"	19·023, ·08365,	1·7854, ·713926,	736·93072, 8327·591,	15·391, ·00086, "	9101·519556
6.	"	31·01, 16·310,	162·718, ·38279,	·037, ·00615,	8·6195, 27·382, "	246·46544

SUBTRACTION OF DECIMALS.

Rule.—Write down the numbers so that the points may be under one another, as in Addition; then proceed as in Simple Subtraction. When there are not as many figures in the upper as in the under line, *nothings* are supposed to be annexed to the former.

Example.—From 643·157
take 29·76231
613·39469 *Answer.*

Exercises.

1.	From	9·267	take	6·7203,	Ans.	2·5467
2.	"	14·796	"	12·605,	"	2·191
3.	"	19·876	"	3·042361,	"	16·833639
4.	"	17·96432	"	12·3745,	"	5·58982
5.	"	11	"	3·16847,	"	7·83153
6.	"	316·281	"	30·379624,	"	285·901376

MULTIPLICATION OF DECIMALS.

Rule.—Write down the multiplicand and multiplier without attending to the points, and multiply as in Simple Multiplication; then point off from the product as many decimals as are contained in *both* quantities: if the product does not contain as many, *prefix* nothings to make up the required number.

Examples.—Multiply 3·061 by 2·5, and ·2312 by ·021.

(1.) 3061
25
15305
6122
7·6525 *Ans.*

In No. 1, there being four decimals in the two quantities, four decimals are pointed off in the answer.

In No. 2, there being seven decimals in the two quantities, and only five in the product, two nothings must be prefixed to make up the number.

(2.) ·2312
·021
2312
4624
·0048552 *Ans.*

Note.—To Multiply by 10, 100, or 1 with any other number of nothings annexed, it is only necessary to remove the decimal point as many places to the *right*, as there are nothings in the multiplier, thus—46·78 multiplied by 10, becomes 467·8.

Exercises.

1.	Multiply	46·78	by	2·3,	Ans.	107·594
2.	"	321·76	"	5·42,	"	1743·9392
3.	"	45·021	"	·023,	"	1·035483
4.	"	1·3215	"	·0051,	"	·00673965
5.	"	97·236	"	10,	"	972·36
6.	"	154·321	"	100,	"	15432·1
7.	"	274·93857	"	·0283,	"	7·780761531
8.	"	3·1415967	"	3·795,	"	11·9223594765

DIVISION OF DECIMALS.

RULE.—Divide as in Simple Division, without attending to the points: then point off as many decimals in the answer, as the dividend contains *more* than the divisor. If the quotient has not as many figures as will allow of this, *prefix* the required nothings to make up the number.

When the dividend has not as many decimals as the divisor, before dividing, annex as many nothings to the dividend as will make the decimals in both equal.

When there is a remainder after dividing, the division may be carried further by annexing nothings to the dividend, which, of course, must be taken into account in pointing off the decimals in the answer.

Example 1.—Divide 3·36 by 2·1.

```
2·1 )3·36( 1·6
     21
     ---
     126
     126
     ---
```

Here one decimal is pointed off in the answer, as the dividend contains one decimal more than the divisor.

Example 2.—Divide 3·36 by ·105.

```
·105 )3·360( 32·
       315
       ---
       210
       210
       ---
```

Before dividing, a nothing is here annexed to the dividend, to make the decimals in the dividend and divisor equal: and being thus equal, there are no decimals in the answer.

Example 3.—Divide ·336 by 21·

```
21· )·336( ·016
      21
      ---
      126
      126
      ---
```

Here the quotient is 16, but as the dividend has three decimals and the divisor none, the answer ought to have three decimals: a nothing, therefore, requires to be prefixed to the quotient, to make up the number.

NOTE.—To DIVIDE by 10, 100, or 1 with any other number of nothings annexed, it is only necessary to remove the decimal point as many places to the *left* as there are nothings in the divisor; thus—124·5 divided by 100, becomes 1·245.

Exercises.

1.	Divide	231·0	by	4·2,	Ans.	55·
2.	"	36·21	"	21·3,	"	1·7
3.	"	7·424	"	·32,	"	23·2
4.	"	124·5	"	·15,	"	830·
5.	"	497·235	"	10,	"	49·7235
6.	"	1284·127	"	100,	"	12·84127
7.	"	16·7235	"	98·7629,	"	·1693297
8.	"	71·23$\dot{7}$	"	·069184,	"	1029·6857

SEE EXERCISES IN DECIMAL MONEY, APPENDIX, page 139.

PRACTICE.

Practice is a short method of calculating the value of goods, &c., by means of fractional or aliquot parts of the price.

One number is said to be an *aliquot part* of another, when it is contained in it an exact number of times: thus 4 is an aliquot part of 16—namely, $\frac{1}{4}$—as it is contained in it exactly 4 times.

The calculations in Practice are performed by means of such tables of aliquot parts as the following, which should be carefully committed to memory.

The Rules of Practice are generally employed instead of those of Compound Multiplication, for calculating the prices of goods, &c., as the working is in many cases easier and shorter.

TABLES OF ALIQUOT PARTS.

MONEY.

10/	=	$\frac{1}{2}$ of £1	3/4	=	$\frac{1}{3}$ of 10/	9*d.*	=	$\frac{1}{10}$ of 7/6
6/8	=	$\frac{1}{3}$ ″	2/6	=	$\frac{1}{4}$ ″	8*d.*	=	$\frac{1}{6}$ ″ 4/
5/	=	$\frac{1}{4}$ ″	″	=	$\frac{1}{2}$ of 5/	″	=	$\frac{1}{5}$ ″ 3/4
4/	=	$\frac{1}{5}$ ″	2/	=	$\frac{1}{5}$ ″ 10/	″	=	$\frac{1}{3}$ ″ 2/
3/4	=	$\frac{1}{6}$ ″	″	=	$\frac{1}{2}$ ″ 4/	″	=	$\frac{1}{2}$ ″ 1/4
2/6	=	$\frac{1}{8}$ ″	1/8	=	$\frac{1}{6}$ ″ 10/	7½*d.*	=	$\frac{1}{8}$ ″ 5/
2/	=	$\frac{1}{10}$ ″	″	=	$\frac{1}{4}$ ″ 6/8	5*d.*	=	$\frac{1}{12}$ ″ 5/
1/8	=	$\frac{1}{12}$ ″	″	=	$\frac{1}{3}$ ″ 5/	″	=	$\frac{1}{8}$ ″ 3/4
1/4	=	$\frac{1}{15}$ ″	″	=	$\frac{1}{2}$ ″ 3/4	″	=	$\frac{1}{6}$ ″ 2/6
1/3	=	$\frac{1}{16}$ ″	1/4	=	$\frac{1}{5}$ ″ 6/8	″	=	$\frac{1}{4}$ ″ 1/8
1/	=	$\frac{1}{20}$ ″	″	=	$\frac{1}{3}$ ″ 4/	4*d.*	=	$\frac{1}{12}$ ″ 4/
6*d.*	=	$\frac{1}{40}$ ″	1/3	=	$\frac{1}{8}$ ″ 10/	″	=	$\frac{1}{10}$ ″ 3/4
6*d.*	=	$\frac{1}{2}$ of 1/	″	=	$\frac{1}{4}$ ″ 5/	″	=	$\frac{1}{6}$ ″ 2/
4*d.*	=	$\frac{1}{3}$ ″	″	=	$\frac{1}{2}$ ″ 2/6	3*d.*	=	$\frac{1}{10}$ ″ 2/6
3*d.*	=	$\frac{1}{4}$ ″	10*d.*	=	$\frac{1}{12}$ ″ 10/	″	=	$\frac{1}{8}$ ″ 2/
2*d.*	=	$\frac{1}{6}$ ″	″	=	$\frac{1}{8}$ ″ 6/8	″	=	$\frac{1}{5}$ ″ 1/3
1½*d.*	=	$\frac{1}{8}$ ″	″	=	$\frac{1}{6}$ ″ 5/	1½*d.*	=	$\frac{1}{4}$ ″ 6*d.*
1*d.*	=	$\frac{1}{12}$ ″	″	=	$\frac{1}{4}$ ″ 3/4	0¾*d.*	=	$\frac{1}{8}$ ″ 6*d.*
0½*d.*	=	$\frac{1}{2}$ of 1*d.*	″	=	$\frac{1}{3}$ ″ 2/6	″	=	$\frac{1}{4}$ ″ 3*d.*
0¼*d.*	=	$\frac{1}{4}$ ″	″	=	$\frac{1}{2}$ ″ 1/8	″	=	$\frac{1}{2}$ ″ 1½*d.*

WEIGHT.

2 qrs.	=	$\frac{1}{2}$ of 1 cwt.	16 lbs.	=	$\frac{1}{7}$ of 1 cwt.	7 lbs.	=	$\frac{1}{16}$ of 1 cwt.
1 qr.	=	$\frac{1}{4}$ ″	14 lbs.	=	$\frac{1}{8}$ ″	″	=	$\frac{1}{8}$ ″ 2 qrs.
14 lbs.	=	$\frac{1}{2}$ of 1 qr.	″	=	$\frac{1}{4}$ of 2 qrs.	″	=	$\frac{1}{2}$ ″ 14 lbs.
7 ″	=	$\frac{1}{4}$ ″	8 lbs.	=	$\frac{1}{14}$ ″ 1 cwt.	4 lbs.	=	$\frac{1}{28}$ ″ 1 cwt.
4 ″	=	$\frac{1}{7}$ ″	″	=	$\frac{1}{7}$ ″ 2 qrs.	″	=	$\frac{1}{14}$ ″ 2 qrs.
3½ ″	=	$\frac{1}{8}$ ″	″	=	$\frac{1}{2}$ ″ 16 lbs.	″	=	$\frac{1}{4}$ ″ 16 lbs.

The same principle of aliquot parts can be applied to any other weights or *measures.*

I. When the given quantity is a simple number, and the price is an aliquot part of £1, or 1*s*., or 1*d*.

Rule.—Consider the quantity as so many pounds, shillings, or pence, according to the aliquot part used: that is, if the price be an aliquot part of a pound, consider the quantity as so many pounds; if part of a shilling, consider the quantity as so many shillings, &c.: then divide the sum as in Compound Division, by 2, if the aliquot part is ½; by 3, if it is ⅓; by 4, if it is ¼; and so on; converting the quotient, when necessary, into its highest denomination.

Example 1.—What is the price of 713 yards of silk, at 3*s*. 4*d*. a yard?

```
          £    £
3/4 = ⅙ )713
         £118 16 8 Ans.
```

Here the quantity is considered as £713, and is divided as in Compound Division, by 6, because 3*s*. 4*d*. is the ⅙th of £1.

Example 2.—What is the price of 1256 yards of cotton, at 4*d*. a yard?

```
          s.   s.
4d. = ⅓ )1256
     2,0 )418  8
          £20 18 8 Ans.
```

Here the quantity is considered as 1256*s*., and is divided by 3, as 4*d*. is ⅓ of a shilling: the quotient is 418*s*. 8*d*., which being converted into its highest denomination, gives £20, 18*s*. 8*d*. for the answer.

Exercises.

Find the value of the following:—

1.	5784	yards at	0¼*d*.	Ans.	£6	0	6
2.	4718	"	0½*d*.	"	9	16	7
3.	939	"	1*d*.	"	3	18	3
4.	1617	"	1½*d*.	"	10	2	1½
5.	2031	"	2*d*.	"	16	18	6
6.	2732	"	3*d*.	"	34	3	0
7.	3621	"	4*d*.	"	60	7	0
8.	973	"	6*d*.	"	24	6	6
9.	1724	"	1*s*.	"	86	4	0
10.	784	"	1*s*. 8*d*.	"	65	6	8
11.	917	"	1*s*. 8*d*.	"	76	8	4
12.	915	"	2*s*.	"	91	10	0
13.	1346	"	2*s*. 6*d*.	"	168	5	0
14.	1845	"	2*s*. 6*d*.	"	230	12	6
15.	2735	"	3*s*. 4*d*.	"	455	16	8
16.	3716	"	4*s*.	"	743	4	0
17.	3786	"	5*s*.	"	946	10	0
18.	3461	"	6*s*. 8*d*.	"	1153	13	4
19.	7234	"	6*s*. 8*d*.	"	2411	6	8
20.	3475	"	10*s*.	"	1737	10	0

II. WHEN THE QUANTITY IS A SIMPLE NUMBER, BUT THE PRICE IS *not* AN ALIQUOT PART OF £1, OR 1*s*., OR 1*d*.

RULE.—Find the nearest or most convenient aliquot part of the price, and divide the quantity as in Rule I.; then take such other parts as will make up the rest of the price, and divide in each case, that sum of which the aliquot part is taken; then add all the quotients together for the answer.

When there are pounds in the price, multiply the quantity by the pounds, and add to the product the value of the aliquot parts.

When the price is near a pound, and the *difference* is an aliquot part of a pound, the given quantity may be reckoned as so many pounds, and the aliquot part for the difference *subtracted* from the amount; the result is the answer.

NOTE.—Before beginning to divide, the sums represented by the different aliquot parts to be used, should be written opposite the parts, and then added together, that it may be seen if the whole of the price has been taken into account.

Example 1.—What is the price of 360 yards at 15*s*.?

		£360
10/	= $\frac{1}{2}$ of £1,	180
5/	= $\frac{1}{2}$ „ 10/	90
15/		£270 *Ans.*

Here we reckon the quantity as £360, and divide it by 2, as 10*s*. is the $\frac{1}{2}$ of £1; and then the quotient by 2, as 5*s*. is the $\frac{1}{2}$ of 10*s*. The answer is £270.

Example 2.—What is the price of 549 yards at 18*s*. 9$\frac{1}{2}$*d*. a yard.

		£549		
10/	= $\frac{1}{2}$ of £1,	274	10	0
5/	= $\frac{1}{2}$ „ 10/	137	5	0
3/4	= $\frac{1}{3}$ „ 10/	91	10	0
5*d*.	= $\frac{1}{8}$ „ 3/4	11	8	9
0$\frac{1}{2}$*d*.	= $\frac{1}{10}$ „ 5*d*.	1	2	10$\frac{1}{2}$
18/9$\frac{1}{2}$		£515	16	7$\frac{1}{2}$ *Ans.*

Here we take the various sums that make up 18*s*. 9$\frac{1}{2}$*d*. (adding them together to see that the sum is correct), and write opposite them the respective aliquot parts that they represent. We then divide the given quantity, which is considered as £549, by 2 for the first aliquot part; the quotient by 2 for the next; and so on; always dividing that sum of which the aliquot part is taken. In the present example, it will be noticed that 3*s*. 4*d*., which is $\frac{1}{3}$ of 10*s*., divides the quotient of 10*s*., and not that which immediately precedes it, because it is $\frac{1}{3}$ of 10*s*. that is taken. When all the aliquot parts have been used for division, the quotients are added together for the answer.

Example 3.—What is the price of 1546 yards at 18*s*. 4*d*.

	£	*yds.*		
1/8 = $\frac{1}{12}$	)	1546		
		128	16	8
		£1417	3	4 *Ans.*

Here if the price were £1 per yard, the answer, it is evident, would be just as many pounds as yards; but as the price is 1*s*. 8*d*. less than a pound—that is, $\frac{1}{12}$th part less—1-12th of the number, considered as pounds, is subtracted from it, and the *difference is the price* of so many yards at 18*s*. 4*d*.

Example 4.—What is the price of 424 cwts. at £2, 7*s*. 6*d*. a cwt.?

		424
		2
		848
5/	= ¼ of £1,	106
2/6	= ½ " 5/	53
7/6		£1007 *Ans.*

Here we first multiply the quantity 424, by the pounds, 2, and then ascertain the price at 7*s*. 6*d*. by taking aliquot parts; the whole is added for the answer.

Exercises.—Find the value of the following:—

1.	2764 yards at	0¾*d*.	Ans.	£8	12	9
2.	3843 "	0¾*d*.	"	12	0	2¼
3.	4536 "	2½*d*.	"	47	5	0
4.	1635 "	3½*d*.	"	23	16	10½
5.	893 "	4½*d*.	"	16	14	10½
6.	4735 "	5*d*.	"	98	12	11
7.	895 "	5½*d*.	"	20	10	2½
8.	1834 "	7½*d*.	"	57	6	3
9.	1732 "	9*d*.	"	64	19	0
10.	3162 "	10½*d*.	"	138	6	9
11.	2763 "	8½*d*.	"	97	17	1½
12.	1782 "	2*s*. 4*d*.	"	207	18	0
13.	2364 "	4*s*. 8*d*.	"	551	12	0
14.	4623 "	5*s*. 7½*d*.	"	1300	4	4½
15.	4768 "	7*s*. 6*d*.	"	1788	0	0
16.	3217 "	8*s*. 4*d*.	"	1340	8	4
17.	5326 "	9*s*.	"	2396	14	0
18.	3172 "	12*s*. 6*d*.	"	1982	10	0
19.	1637 "	13*s*. 4*d*.	"	1091	6	8
20.	2354 "	15*s*.	"	1765	10	0
21.	1859 "	16*s*. 8*d*.	"	1549	3	4
22.	978 "	17*s*. 6*d*.	"	855	15	0
23.	1036 "	18*s*.	"	932	8	0
24.	1384 "	18*s*. 4*d*.	"	1268	13	4
25.	2137 "	19*s*.	"	2030	3	0
26.	217 cwts. at	£1, 8*s*. 4*d*.	"	307	8	4
27.	254 "	£1, 9*s*. 6*d*.	"	374	13	0
28.	187 "	£1, 12*s*. 6*d*.	"	303	17	6
29.	345 "	£1, 16*s*.	"	621	0	0
30.	293 "	£1, 17*s*. 6*d*.	"	549	7	6
31.	317 "	£2, 8*s*. 8*d*.	"	771	7	4
32.	381 "	£2, 14*s*. 6*d*.	"	1038	4	6
33.	537 "	£3, 12*s*. 9*d*.	"	1953	6	9
34.	462 "	£3, 17*s*. 4*d*.	"	1786	8	0
35.	619 "	£4, 5*s*. 10½*d*.	"	2657	16	7½
36.	735 "	£2, 16*s*. 8*d*.	"	2082	10	0
37.	426 "	£5, 9*s*. 8*d*.	"	2335	18	0
38.	387 "	£4, 11*s*. 8*d*.	"	1773	15	0
39.	314 "	£6, 14*s*. 4*d*.	"	2109	0	8
40.	437 "	£3, 6*s*. 9¾*d*.	"	1459	17	0¾

III. When the quantity contains a Fraction.

Rule.—Calculate the value of the integer by either of the preceding rules; then multiply the given price by the fraction,* and add the whole for the answer.

* See Compound Multiplication, Rule V. page 48.

Example.—What is the value of 3276¾ yards at 5*s.*?

	£			
5/ = ¼	)3276			
	819			
	0	3	9	
	£819	3	9	*Ans.*

Here we calculate the value of the 3276 yards; then separately the value of ¾ yards; and add both sums for the answer.

5*s.*
3
4) 15
3*s.* 9*d.*

Exercises.

Find the value of the following:—

1.	839¼	cwts. at	£0, 18*s.* 4*d.*	Ans. £769	6	3	
2.	917½	〃	£1, 4*s.* 6*d.*	〃 1123	18	9	
3.	1018¾	〃	£1, 11*s.* 6*d.*	〃 1604	10	7½	
4.	1737⅗	〃	£2, 6*s.* 9*d.*	〃 4061	8	5¼	⅜
5.	2364⅜	〃	£2, 14*s.* 8*d.*	〃 6462	12	6	
6.	3185 4/7	〃	£3, 13*s.* 6*d.*	〃 11706	19	6	
7.	2873⅝	〃	£2, 16*s.* 8*d.*	〃 8141	18	9	
8.	3657⅚	〃	£4, 18*s.* 4*d.*	〃 17984	6	11¼	⅓
9.	1789 3/7	〃	£1, 15*s.* 6*d.*	〃 3176	4	8½	2/7
10.	876 7/9	〃	£1, 17*s.* 3*d.*	〃 1632	19	11½	⅔
11.	1234⅜	〃	£1, 11*s.* 11¼*d.*	〃 1971	2	10¾	⅞
12.	7291⅘	〃	£1, 19*s.* 9¾*d.*	〃 14514	10	7½	⅓

IV. When the quantity consists of several denominations.

Rule.—Multiply the price by the highest denomination of the quantity, and for the other denominations take aliquot parts of the quantity, and proportionate aliquot parts of the price.

Example.—What is the value of 7 cwts. 3 qrs. 7 lbs. at £5, 12*s.* 6*d.* per cwt.?

cwts.	*qrs.*	*lbs.*					
				£5	12	6	
7	0	0 =				7	
				39	7	6	
0	2	0 =	½ cwt.	2	16	3	
0	1	0 =	½ of 2 qrs.	1	8	1½	
0	0	7 =	¼ 〃 1 qr.	0	7	0¼ 2/4	
7	3	7		£43	18	10¾ 2/4	*Ans.*

Here the price, £5, 12*s.* 6*d.* is multiplied by the 7 cwts.; then for 2 qrs. the price of ½ cwt. is taken; for 1 qr. the half of the price of 2 qrs.; and for 7 lbs. the ¼ of the price of 1 quarter.

Exercises.—Find the value of the following:—

1.	9 cwts. 1 qr. 14 lbs.	at £2, 8*s*. 6*d*.	Ans. £22	14	$8\frac{1}{4}$	
2.	15 " 0 " 16	" £1, 18*s*. 9*d*.	" 29	6	$9\frac{1}{4}$	$\frac{5}{7}$
3.	21 " 2 " 8	" £3, 5*s*. 8*d*.	" 70	16	$6\frac{1}{4}$	$\frac{1}{7}$
4.	28 " 3 " 6	" £3, 14*s*. 7*d*.	" 107	8	$3\frac{2}{4}$	$\frac{11}{14}$
5.	25 qrs. 4 bush. 2 pks.	at £3, 15*s*. 6*d*.	" 96	9	$11\frac{1}{2}$	$\frac{1}{2}$
6.	76 yds. 3 qrs. 2 nails	" £0, 14*s*. 6*d*.	" 55	14	$8\frac{1}{4}$	
7.	136 acres 3 ro. 20 per.	" £1, 9*s*. 6*d*.	" 201	17	$9\frac{3}{4}$	
8.	273 gallons 6 pints	" £0, 15*s*. 8*d*.	" 214	8	9	

V. TO FIND THE VALUE, BY MULTIPLYING THE QUANTITY BY THE PRICE.

RULE.—If the price be *pence*, consider the quantity as so many pence; or if *shillings*, consider the quantity as so many shillings; and then multiply by the number of pence or shillings: if the price be shillings and pence, multiply by the shillings, and by a proportionate fraction for the pence; or if pence and farthings, multiply by the pence, and by a fraction for the farthings.

NOTE.—If the price be an *even* number of shillings, multiply the quantity by half the number; then double the last figure of the product for the shillings of the answer, and the rest of the figures are the pounds: thus, to find the price of 426 yards at 4*s*., multiply 426 by 2, making 852; then double the last figure for the shillings of the answer, and the other figures are the pounds: the answer thus being £85, 4*s*.

Examples.—What is the cost of 112 lbs. at 7*d*. and at 3*s*. 6*d*.?

(1.)

d.		*s*.	*d*.
112	=	9	4
			7
		£3 5	4 *Ans*.

In No. 1, we consider the 112 as 112*d*. or 9*s*. 4*d*., and then multiply by 7.

In No. 2, we reckon the 112 as 112*s*., and multiply by $3\frac{1}{2}$; the 3 representing the shillings, and the $\frac{1}{2}$, the 6*d*.

(2.)

s.		£	*s*.	*d*.
112	=	5	12	0
				$3\frac{1}{2}$
		16	16	0
		2	16	0
		£19	12	0 *Ans*.

Exercises.—What is the cost of the following?—

1.	1 cwt. of sugar,	at	8*d*. per lb.	Ans. £3	14	8
2.	1 " "		$9\frac{1}{2}$*d*. "	" 4	8	8
3.	60 lbs. of soap,	"	5*d*. "	" 1	5	0
4.	87 " starch,	"	$10\frac{1}{2}$*d*. "	" 3	16	$1\frac{1}{2}$
5.	100 " candle,	"	$8\frac{3}{4}$*d*. "	" 3	12	11
6.	138 yards of calico,	"	$5\frac{1}{4}$*d*. per yard,	" 3	0	$4\frac{1}{2}$
7.	123 " silk,	"	7*s*. "	" 43	1	0
8.	150 " "	"	3*s*. "	" 22	10	0
9.	186 " of linen,	"	2*s*. 3*d*. "	" 20	18	6
10.	246 " "	"	3*s*. 9*d*. "	" 46	2	6
11.	47 " of cloth,	"	8*s*. "	" 18	16	0
12.	59 " "	"	6*s*. "	" 17	14	0
13.	75 " "	"	12*s*. "	" 45	0	0
14.	105 " "	"	18*s*. "	" 94	10	0

Miscellaneous Exercises in Practice.

			£	s.	d.
1.	Find the value of 964 yards at 2s. 9d.	Ans.	£ 132	11	0
2.	″ ″ 567 lbs. at £1, 7s. 6d.	″	779	12	6
3.	″ ″ 4351 cwts. at 5s. 6d.	″	1196	10	6
4.	″ ″ 9327 qrs. at 2s. 4d.	″	1088	3	0
5.	″ ″ 764 yards at 3½d.	″	11	2	10
6.	″ ″ 3997 ″ 7s. 9d.	″	1548	16	9
7.	″ ″ 482 cwts. at £7, 2s. 3d.	″	3428	4	6
8.	″ ″ 9647 oz. at 15s. 6d.	″	7476	8	6
9.	″ ″ 4325 gals. at £1, 16s. 8d.	″	7929	3	4
10.	″ ″ 189 yards at £3, 4s. 7d.	″	610	6	3
11.	″ ″ 3716 lbs. at 7½d.	″	116	2	6
12.	″ ″ 2943 oz. at 9d.	″	110	7	3
13.	″ ″ 3916 qrs. at 3s. 9d.	″	734	5	0
14.	″ ″ 475 gals. at £5, 6s. 8d.	″	2533	6	8
15.	″ ″ 968 cwts. at £3, 2s. 4d.	″	3016	18	8
16.	″ ″ 1234 ″ 6¾d.	″	34	14	1½
17.	″ ″ 5678 tons at 1s. 9½d.	″	508	13	1
18.	″ ″ 9012 ″ £7, 15s. 10d.	″	70218	10	0
19.	″ ″ 751 lbs. at £1, 12s. 6d.	″	1220	7	6
20.	″ ″ 3964 acres at 8s. 8d.	″	1717	14	8
21.	″ ″ 1009 ″ £1, 5s. 10d.	″	1303	5	10
22.	″ ″ 374 yards at 5½d.	″	8	11	5
23.	″ ″ 2166 tons at 6s. 9¾d.	″	737	15	10½
24.	″ ″ 2772 cwts. at £2, 7s. 5¼d.	″	6574	16	9
25.	″ ″ 4165 qrs. at £3, 1s. 11½d.	″	12902	16	5¼
26.	″ ″ 1893 lbs. at 2s. 10¾d.	″	274	1	9¾
27.	″ ″ 2714 oz. at £1, 16s. 11d.	″	5009	11	10
28.	″ ″ 1862 yards at £2, 3s. 5¾d.	″	4047	18	2½
29.	″ ″ 3931 ″ 2s. 11d.	″	573	5	5
30.	″ ″ 5624 ″ £1, 15s. 9½d.	″	10064	12	4
31.	″ ″ 793 lbs. at £2, 6s. 10¾d.	″	1859	8	4¾
32.	″ ″ 125 cwts. at 17s. 5½d.	″	109	2	3½
33.	″ ″ 937 tons at 19s. 6d.	″	913	11	6
34.	″ ″ 1824 qrs. at 13s. 4¾d.	″	1221	14	0
35.	″ ″ 2793 oz. at £2, 1s. 3½d.	″	5766	7	7½
36.	″ ″ 1395 acres at £1, 13s. 11¼d.	″	2367	2	9¾
37.	″ ″ 2068 yards at 12s. 10½d.	″	1331	5	6
38.	″ ″ 1754 tons at £1, 15s. 11d.	″	3149	17	10
39.	″ ″ 8967 cwts. at 6s. 9¾d.	″	3054	7	8¼
40.	″ ″ 391 yards at £5, 8s. 6½d.	″	2121	19	9½
41.	″ ″ 7184 barrels at 5s. 10½d.	″	2110	6	0
42.	″ ″ 327½ yards at £1, 2s. 7½d.	″	370	9	8¼
43.	″ ″ 756⅚ cwts. at £3, 18s. 6d.	″	2970	11	5
44.	″ ″ 742 cwts. 1 qr. 23 lbs. at £3, 19s. 4d. per cwt.	″	2945	1	5½
45.	″ ″ 12 ac. 3 ro. 25 po. at £103, 13s. 11d. per acre.	″	1338	6	5¾ ⅜

1. What is the value of a carpet, measuring 46 yards, at 3*s.* 4*d.* a yard? . . . . Ans. £7, 13*s.* 4*d.*

2. What will a piece of linen, measuring 54 yards, amount to at 2*s.* 8*d.* a yard? . . . . Ans. £7, 4*s.*

3. Bought 2136 lbs. of sugar, at $7\frac{1}{2}$*d.* per lb.; what did it amount to? . . . . Ans. £66, 15*s.*

4. Sold 3564 yards of calico, at 8*d.* a yard; how much did I receive? . . . . Ans. £118, 16*s.*

5. What is the price of 2 cwts. of soap, at $5\frac{1}{4}$*d.* per lb.? Ans. £4, 18*s.*

6. Bought 2 pieces of linen sheeting, each measuring $54\frac{3}{4}$ yards, at 2*s.* 2*d.* a yard; what did I pay for the whole? Ans. £11, 17*s.* 3*d.*

7. What is the value of a chest of tea containing $97\frac{1}{2}$ lbs. at 4*s.* 3*d.* per pound? . . . . . . Ans. £20, 14*s.* $4\frac{1}{2}$*d.*

8. What is the value of 15 dozen pair of shoes, at 4*s.* 8*d.* a pair? . . . . Ans. £42.

9. Sold 6 dozen pair of fine gloves, at 2*s.* 3*d.* a pair; what did the whole amount to? . . . . Ans. £8, 2*s.*

10. If the carriage of goods from Edinburgh to Glasgow, per canal, be 1*s.* 9*d.* per cwt., what must be paid for the conveyance of 3 tons 7 cwts.? . . . . Ans. £5, 17*s.* 3*d.*

11. A shopkeeper bought 15 pieces of plain ribbon, each piece consisting of 36 yards, at $6\frac{3}{4}$*d.* a yard; how much did he pay for the whole? . . . . Ans. £15, 3*s.* 9*d.*

12. What will 27 dozen pair of stockings come to, at 1*s.* 9*d.* a pair? . . . . Ans. £28, 7*s.*

13. What is the cost of 272 cwts. 1 qr. 21 lbs. of cheese, at £3, 2*s.* $3\frac{3}{4}$*d.* per cwt.? . . . . . Ans. £848, 16*s.* $3\frac{0}{4}$*d.* $\frac{9}{16}$

14. What is the cost of 123 acres 2 roods 20 poles of land, at £102, 13*s.* 6*d.* per acre? . . . . Ans. £12,693, 3*s.* $11\frac{1}{4}$*d.*

15. What is the value of 239 quarters 5 bushels of barley, at £1, 13*s.* 7*d.* per quarter? . . . . . Ans. £402, 7*s.* $4\frac{3}{4}$*d.* $\frac{1}{2}$

16. If a cwt. of Dutch salt cost £4, 7*s.* $11\frac{1}{2}$*d.*, how much have I to pay for 213 cwts. 3 qrs. 14 lbs.? . Ans. £940, 12*s.* $1\frac{0}{4}$*d.* $\frac{1}{4}$

17. What is the value of 723 feet 11 inches of mahogany, at 3*s.* $2\frac{1}{2}$*d.* per foot? . . . . Ans. £116, 2*s.* $6\frac{3}{4}$*d.* $\frac{1}{6}$

18. What is the price of 549 qrs. of wheat, at £2, 7*s.* 6*d.* per quarter? . . . . Ans. £1303, 17*s.* 6*d.*

19. What is the value of $725\frac{3}{8}$ cwts. of raisins, at £3, 12*s.* 9*d.* per cwt.? . . . . Ans. £2638, 11*s.* $0\frac{1}{4}$*d.* $\frac{1}{2}$

20. What is the cost of 123 cwts. 2 qrs. 7 lbs. of hemp, at £1, 5*s.* 6*d.* per cwt.? . . . . . Ans. £157, 10*s.* $10\frac{0}{4}$*d.* $\frac{1}{2}$

MENTAL ARITHMETIC.

MENTAL ARITHMETIC is the term applied to certain expeditious methods of calculating in the *mind* alone, without aid from either pen or pencil. A large portion of the common calculations occurring in business, are thus performed entirely in the mind. It would be impossible to give all the rules for such calculations, as they are so various: the following, however, are those most generally employed:—

I. TO MULTIPLY PENCE AND FARTHINGS BY ONE DOZEN.

RULE.—Reckon every penny in the given sum as 1*s.*, and every farthing as 3*d.*

The price of 3, or 6, may be found by taking the ¼ or ½ of the price of 13.

Exercises.—Multiply the following by 12 :—

1.	1¼*d.*	Ans. £0	1	3		7.	9½*d.*	Ans. £0	9	6
2.	2½*d.*	〃	0	2	6	8.	10¼*d.*	〃	0	10	3
3.	3¾*d.*	〃	0	3	9	9.	10¾*d.*	〃	0	10	9
4.	4½*d.*	〃	0	4	6	10.	11¼*d.*	〃	0	11	3
5.	7¾*d.*	〃	0	7	9	11.	11½*d.*	〃	0	11	6
6.	8¼*d.*	〃	0	8	3	12.	11¾*d.*	〃	0	11	9

II. TO MULTIPLY PENCE AND FARTHINGS BY ANY NUMBER OF DOZENS.

RULE.—Find the price of *one* dozen by Rule I.; then multiply by 2, 3, &c. according to the number of dozens.

Thus to find the price of 72 at 2½*d.* each (6 dozen), multiply 2*s.* 6*d.* the price of 1 dozen, by 6, and the answer is 15*s.*

Exercises.—Multiply the following :—

1. 2½*d.* by 24,	Ans. £0	5	0	5. 10½*d.* by 60,	Ans. £2	12	6
2. 8¼*d.* 〃 24,	〃 0	16	6	6. 7¾*d.* 〃 72,	〃 2	6	6
3. 9¾*d.* 〃 36,	〃 1	9	3	7. 8½*d.* 〃 84,	〃 2	19	6
4. 10¼*d.* 〃 48,	〃 2	1	0	8. 11¼*d.* 〃 96,	〃 4	10	0

III. TO MULTIPLY PENCE AND FARTHINGS by 100.

RULE.—Convert the given price to farthings; then reckon the quantity as so many pence, and twice the quantity as so many shillings.

Thus, to multiply 1½*d.* by 100, convert the 1½*d.* to farthings—namely, 6—and 6 will stand for the pence, and twice 6, or 12, for the shillings of the answer; which will thus be 12*s.* 6*d.*

Exercises.—Multiply the following by 100.

1.	1¾*d.*	Ans. £0	14	7	4.	7½*d.*	Ans. £3	2	6
2.	2½*d.*	〃 1	0	10	5.	8¾*d.*	〃 3	12	11
3.	5¾*d.*	〃 2	7	11	6.	10¾*d.*	〃 4	9	7

IV. To Multiply Pence and Farthings by 112, the number of pounds in a hundredweight.

Rule.—Multiply 9*s*. 4*d*. (112*d*.) by the number of pence, and add 2*s*. 4*d*. for every farthing.

Thus, to find the price of 112 lbs. at 2½*d*. a lb., multiply 9*s*. 4*d*. by 2, adding 4*s*. 8*d*. for the ½*d*., and the answer is £1, 3*s*. 4*d*.

Exercises.—What is the price of 112 lbs. at—

1.	1¾*d*. . . Ans.	£0	16	4	4.	6½*d*. . . Ans.	£3	0	8
2.	2¼*d*. . . 〃	1	1	0	5.	8¼*d*. . . 〃	3	17	0
3.	3*d*. . . . 〃	1	8	0	6.	9¾*d*. . . 〃	4	11	0

V. To Multiply Pence and Farthings by 52, the number of weeks in a year.

Rule.—Multiply 4*s*. 4*d*. (52*d*.) by the number of pence, and add 1*s*. 1*d*. for every farthing.

Thus, to find the price of 52 at 3½*d*., multiply 4*s*. 4*d*. by 3, adding 2*s*. 2*d*. for the ½*d*., and the answer is 15*s*. 2*d*.

Exercises.—Multiply the following by 52 :—

1.	4¼*d*. . . Ans.	£0	18	5	4.	8½*d*. . . Ans.	£1	16	10
2.	6¾*d*. . . 〃	1	9	3	5.	9¼*d*. . 〃	2	0	1
3.	7½*d*. . . 〃	1	12	6	6.	10¾*d*. . . 〃	2	6	7

VI. To Multiply Pence and Farthings by 313, the number of working-days in a year.

Rule.—Multiply £1, 6*s*. 1*d*. (313*d*.) by the number of pence, and add 6*s*. 6¼*d*. for every farthing.

Thus, to find the price of 313 at 2¼*d*., multiply £1, 6*s*. 1*d*. by 2, adding 6*s*. 6¼*d*. for the ¼*d*., and the answer is £2, 18*s*. 8¼*d*.

Exercises.—What is the price of 313, at—

1.	3½*d*. . . Ans.	£4	11	3½	4.	7¾*d*. . Ans.	£10	2	1¾
2.	5*d*. . . 〃	6	10	5	5.	8½*d*. 〃	11	1	8½
3.	6½*d*. . . 〃	8	9	6½	6.	11*d*. . 〃	14	6	11

VII. To Multiply Pence and Farthings by 365, the number of days in a year.

Rule.—Multiply £1, 10*s*. 5*d*. (365*d*.) by the number of pence, and add 7*s*. 7¼*d*. for every farthing.

Thus, to find the price of 365 at 2½*d*., multiply £1, 10*s*. 5*d*. by 2, adding 15*s*. 2½*d*. for the ½*d*., and the answer is £3, 16*s*. 0½*d*.

Exercises.—What is the price of 365, at—

1.	3¾*d*. . . Ans.	£5	14	0¾	4.	7½*d*. . Ans.	£11	8	1½
2.	5*d*. . . 〃	7	12	1	5.	9¼*d*. 〃	14	1	4¼
3.	6½*d*. . . 〃	9	17	8½	6.	10½*d*. . 〃	15	19	4½

VIII. To Multiply Shillings and Pence by 20.

Rule.—Reckon the shillings as so many pounds, and every 3*d.* as 5*s.*; or for every penny 1*s.* 8*d.*

Exercises.

1. What is the value of 20 stuff hats, at 15*s.* each? Ans. £15.
2. " " value of 20 copies of a book, at 6*s.* each? " £6.
3. " " price of a ton of ore, at 8*s.* 3*d.* per cwt.? " £8, 5*s.*
4. " " price of a ton of lead, at 17*s.* 6*d.* per cwt.? " £17, 10*s.*

IX. To Multiply Shillings and Pence by 52.

Rule.—Multiply £2, 12*s.* (52*s.*) by the number of shillings, and add 4*s.* 4*d.* for every penny.

Exercises.—How much do the following sums per week amount to in a year?

1.	4*s.* . . .	Ans.	£10	8	0		4.	6*s.* . . .	Ans.	£15	12	0
2.	5*s.* .	"	13	0	0		5.	7*s.* 6*d.*	"	19	10	0
3.	5*s.* 9*d.*	"	14	19	0		6.	8*s.* . . .	"	20	16	0

X. To Calculate the price from the nearest round number.

Rule.—Find the value of the given quantity, at the price in round numbers nearest to the real price; then deduct or add the difference between the real price and the round number taken.

Thus, 40 yards at 5½*d.* may be reckoned as 40 at 6*d.* = 20*s.*; and then deducting 40 at ½*d.* = 1*s.* 8*d.* for the difference, the answer is 18*s.* 4*d.*

Exercises.

1. What is the amount of 38 times 7*d.*? . . . Ans. £1, 2*s.* 2*d.*
2. " " 17 " 11½*d.*? . " 16*s.* 3½*d.*
3. How much are 9 pair of gloves, at 1*s.* 5½*d.* a pair? " 13*s.* 1½*d.*
4. " " 18 yards of cloth, at 7*s.* 11*d.* a yard? " £7, 2*s.* 6*d.*

XI. To Divide any number of Pounds by 20.

Rule.—Reckon a shilling for every pound in the price, and threepence for every five shillings.

Exercises.

1. Divide by 20, £50, ... Ans. £2, 10*s.*
2. " " £80, ... " £4.
3. " " £90, 10*s.* . . . . . . " £4, 10*s.* 6*d.*

XII. To Divide any number of Shillings by 12.

Rule.—Reckon a penny for every shilling, and a farthing for every threepence.

Exercises.

1. Divide by 12, 8*s.* 3*d.* . . . . Ans. 0*s.* 8¼*d.*
2. " " 7*s.* 6*d.* . . . . " 0*s.* 7½*d.*
3. " " 30*s.* . . . . " 2*s.* 6*d.*
4. " " 14*s.* 9*d.* . . . . " 1*s.* 2¾*d.*

COMMERCIAL ALLOWANCES.

In buying and selling goods by weight, there are usually certain allowances or deductions made for the weight of the boxes, packages, &c. which contain the goods.

Gross weight, is the total weight of the goods, and boxes or casks, &c. in which they are packed.

Tare, is the weight of the boxes, &c. containing the goods, which is to be deducted from the gross.

Nett weight, is what remains after the tare has been deducted from the gross.

Draft, is a deduction made on some goods, to allow for the loss of weight in selling by retail; it is deducted from the gross weight.

Several other allowances were formerly made, but they are nearly out of use.

The following example will shew the mode of deducting the tare from a quantity of goods:—

Example.—What is the nett weight of 5 tierces of coffee; the first weighing 5 cwts. 2 qrs. 17 lbs., and tare, 2 qrs. 11 lbs.; the second, 6 cwts. 1 qr. 23 lbs., and tare, 3 qrs. 1 lb.; the third, 5 cwts. 3 qrs. 15 lbs., and tare, 2 qrs. 27 lbs.; the fourth, 6 cwts. 3 qrs. 19 lbs., and tare, 3 qrs. 20 lbs.; and the fifth, 6 cwts. 2 qrs. 4 lbs., and tare, 3 qrs. 21 lbs.?

	Gross weight.			Tare.		
	cwts.	*qrs.*	*lbs.*	*cwts.*	*qrs.*	*lbs.*
1.	5	2	17	0	2	11
2.	6	1	23	0	3	1
3.	5	3	15	0	2	27
4.	6	3	19	0	3	20
5.	6	2	4	0	3	21
	31	1	22	3	3	24
Subtract tare,	3	3	24			
Nett weight,	27	1	26			

Exercises.

1. What is the nett weight of 3 barrels of figs, weighing 6 cwts. 2 qrs. 4 lbs., the tare allowed per barrel being 29 lbs.?

Ans. 5 cwts. 3 qrs. 1 lb.

2. What is the nett weight of 7 hogsheads of sugar, each weighing 12 cwts. 1 qr. 25 lbs.; draft, 2 lbs. per hhd.; and tare, 1 cwt. 2 qrs. 27 lbs. per hhd.? . . . . . . Ans. 75 cwts.

3. Find the nett weight of 5 bags of rice, the first weighing 1 cwt. 23 lbs.; second, 1 cwt. 1 qr. 11 lbs.; third, 1 cwt. 1 qr. 22 lbs.; fourth, 1 cwt. 2 qrs.; and fifth, 1 cwt. 2 qrs. 17 lbs.; tare, 20 lbs. per cwt.; and draft, 2 lbs. per bag: and its value at 18*s.* 7½*d.* per cwt. Ans. 5 cwts. 3 qrs. 6 lbs.; £5, 8*s.* 1*d.* nearly.

PERCENTAGES.

PER-CENTAGES are calculations in Interest, Discount, Commission, &c. at given rates per 100. The term is derived from the Latin words *per*, signifying *by*, and *centum* (contracted into cent.), signifying *hundred*, and means *by the hundred*. Thus, 5 per cent. means 5 for every hundred.

TABLE OF PERCENTAGES IN MONEY.

$\frac{1}{8}$ per cent.	=	$\frac{3}{10}d.$ per £1	5 per cent.	=	1/ per £1
$\frac{1}{4}$ "	=	$\frac{3}{5}d.$ "	$7\frac{1}{2}$ "	=	1/6 "
$\frac{1}{2}$ "	=	$1\frac{1}{5}d.$ "	10 "	=	2/ "
1 "	=	$2\frac{2}{5}d.$ "	$12\frac{1}{2}$ "	=	2/6 "
2 "	=	$4\frac{4}{5}d.$ "	25 "	=	5/ "
$2\frac{1}{2}$ "	=	6*d.* "	$33\frac{1}{3}$ "	=	6/8 "
3 "	=	$7\frac{1}{5}d.$ "	50 "	=	10/ "
4 "	=	$9\frac{3}{5}d.$ "	Cent. per cent.	=	20/ or double.

Questions of percentage, whether in regard to money or quantities of any other kind, are virtually wrought by the rules of PROPORTION; the word *cent.* being expressed as 100, in stating the questions. The calculations termed Interest, Discount, Commission, Insurance, &c. are questions of percentage, and are treated under these different heads; but a few examples are given here to shew the general nature of percentages, and the mode of working them.

Example 1.—A gas company reduces the price of its gas from 8*s.* 6*d.* to 7*s.* 6*d.* per 1000 cubic feet; what is the reduction per cent.?

```
 s. d.       £        s.
 8  6  :    100  ::   1
12           20
---        ----
102        2000
             12
     102 )24000
      20 )235  3½
  Ans.  £11 15 3½ per cent.
```

Here we say, as 8*s.* 6*d.* is to £100, so is 1*s.* (the difference between the two rates of charge) to the percentage sought.

Example 2.—A gentleman wishes to buy an estate which produces a rental of £700 per annum; what sum must he pay, in order that the £700 will be equal to $3\frac{1}{2}$ per cent. on his outlay?

```
£  s.      £          £
3  10  :  700   ::   100
20          20
--       -----
70       14000
           100
   7,0 )140000,0
 Ans. £20,000
```

Here we say, as £3, 10*s.* the percentage, is to £700, the rent, so is £100 to the required sum.

Example 3.—A gentleman paid £176 of income-tax. The tax was at the rate of 3 per cent. on his income; what was the amount of his income?

£		£		£
3	:	100	::	176
				100
			3	)17600
			Ans.	£5866 13 4 income.

This example is stated in the order in which it may be expressed in words, according to the rule given at page 59: thus—'If £3 is the tax on £100, on what sum is £176 the tax?'

Exercises.

1. What is the amount of a tax of 6 per cent. on a rental of £24? . . . . Ans. £1, 8*s*. $9\frac{1}{2}$*d*. $\frac{2}{5}$

2. If the charge for conveyance in a stage-coach be lowered from £2, 2*s*. to £1, 15*s*. what reduction is that per cent.? Ans. £16, 13*s*. 4*d*.

3. If a house yield an annual rental of £60, what should I pay for it, in order to clear £$4\frac{1}{2}$ per cent. on the outlay? Ans. £1333, 6*s*. 8*d*.

4. What should I give for the same house to clear £6 per cent.? . . . . Ans. £1000.

5. A person does business to the extent of £2000 per annum, and pays a rent of £82; what percentage is this rental on the £2000? . . . . Ans. $4\frac{1}{10}$ per cent.

6. If a person pays a tax of £8, at 3 per cent. on his income, what is his income? . . . . . . Ans. £266, 13*s*. 4*d*.

7. The receipts of a railway company in 1841 were £997, and in 1842, £1232; what was the increase per cent.? Ans. £23, 11*s*. $4\frac{3}{4}$*d*. $\frac{881}{997}$

8. In 1841, wool was exported to the value of £555,620, and in 1842, the value of what was exported was £510,965; what was the decrease per cent.? . . . . . Ans. $8\frac{79}{2137}$ per cent.

9. Bought paper at 15*s*. a ream, and sold it at 16*s*.; what was gained per cent.? . . . . Ans. £6, 13*s*. 4*d*.

10. Sold cloth at 18*s*. a yard, and lost 5 per cent. upon it; what did it cost? . . . . . . Ans. 18*s*. $11\frac{1}{4}$*d*. $\frac{9}{19}$ a yard.

11. What must I pay to insure property to the amount of £4000, at the rate of $1\frac{1}{2}$ per cent.? . . . . . Ans. £60.

12. A water company increases its charge from 10*d*. to 1*s*. in the pound of rent; what is the increase per cent.? Ans. 20 per cent.

SIMPLE INTEREST.

INTEREST is the sum paid for the use of money, by the person who borrows it to the person who lends it: the sum lent is called the *principal*, and the allowance for lending it, the *interest*.

Interest is calculated at the rate of so much per cent. for a year, and the words *per annum*, meaning *by the year*, are either expressed or understood. By 5 per cent. per annum is signified a charge of £5 for the loan of £100 for a year.

Calculations of yearly interest refer to 365 days, and calculations of interest for shorter periods, are either made on the exact number of days from the day of lending to the day of paying, or on the number of months.

A common rate of interest on ordinary transactions is £5 per cent. but it varies from 2 to 5 per cent.

It is termed *Simple* Interest when charged on the principal only; and *Compound*, when the interest is added to the principal, and then interest charged on the amount of both.

I. To Find the Interest on a given sum for *one* year.

RULE.—Multiply the principal by the rate per cent. and divide the product by 100.

NOTE.—This is also the rule by which any given per cent. is calculated, when no particular period is specified, as in the question, 'How much is 4 per cent. on £150?'

Example.—What is the interest on £325, 10*s.* for one year, at $2\frac{1}{2}$ per cent.?

£	s.	
325	10	
	$2\frac{1}{2}$	
651	0	
162	15	
8,13	15	
	20	
	2,75	
	12	
	9,00	Interest £8 2 9

Here, after multiplying the principal by $2\frac{1}{2}$, the rate per cent. we divide the product by 100, as in Compound Division, Rule III. page 52.

Exercises.—What is the interest on the following sums for a year:—

1.	£185	0	0	at	2	per cent.	Ans. £3	14	0	
2.	240	10	0	〃	$2\frac{1}{2}$	〃	〃 6	0	3	
3.	175	6	8	〃	3	〃	〃 5	5	$2\frac{1}{4}$	$\frac{3}{5}$
4.	310	9	8	〃	$3\frac{1}{2}$	〃	〃 10	17	$4\frac{0}{4}$	$\frac{6}{25}$
5.	295	14	10	〃	4	〃	〃 11	16	$7\frac{0}{4}$	$\frac{12}{25}$
6.	751	15	0	〃	5	〃	〃 37	11	9	

k

INTEREST for a year, may also be readily calculated by taking the interest on £1, according to the percentage table, page 94, and multiplying it by the number of pounds in the given sum.

Thus, to find the interest on £22, at $2\frac{1}{2}$ per cent., multiply 6*d*., the interest on £1, by 22, and the answer is 11*s*.: or to find the interest on £37, at 5 per cent., multiply 1*s*., the interest on £1, by 37, and the answer is 37*s*. = £1, 17*s*. When there are shillings or pence in the given sum, take the proportion of the interest for a pound.

INTEREST for a year at $2\frac{1}{2}$ per cent. on any number of *pounds*, may be readily found as follows:—

Cut off the last figure of the principal, and divide the rest of the sum by 4; the quotient is the pounds of the answer: annex to any remainder, the figure that was cut off, and the half of this sum, reckoned as shillings—or if there is no remainder, the half of the figure that was cut off—is the shillings and pence of the answer.

This process is merely a short way of dividing by 40—as $2\frac{1}{2}$ is the fortieth part of 100.

Example.—What is the interest on £437, at $2\frac{1}{2}$ per cent.?

```
     £
 4 ) 43,7
   £10 18  6
```

Here we cut off the 7, and on dividing 43 by 4, the answer is £10, and 3 over. Annex to the 3 the figure cut off, 7, making 37, counted as 37*s*. and the half of this is 18*s*. 6*d*. The interest, therefore, is £10, 18*s*. 6*d*.

INTEREST for a year at 5 per cent. on any number of *pounds*, may be found as follows:—

Cut off the last figure of the principal, and divide the rest of the sum by 2; the quotient is the pounds of the answer: if there is 1 over, annex to it the figure cut off, and this sum is the shillings of the answer; or if there is no remainder, the figure cut off is the shillings of the answer.

This process is merely a short way of dividing by 20—as 5 is the twentieth part of 100.

Example.—What is the interest on £276, at 5 per cent.?

```
     £
 2 ) 27,6
   £13 16
```

Here we cut off the 6, and on dividing by 2, the answer is £13, and 1 over: annex the 1 to the 6 cut off, making 16, which is the shillings of the answer.

II. To find the Interest for any number of Years.

RULE.—Find the interest for *one* year by Rule I. and then multiply the amount by the number of years.

Example.—What is the interest on £162, 14*s*. 11*d*. for 2 years, at 3 per cent.?

```
            £162  14  11
                       3
      100 ) 488    4   9
              4   17   7¾ 8/100
                       2
Interest,   £9    15   3½ 16/100
```

Here we find by Rule I. that the interest for one year is £4, 17*s*. $7\frac{3}{4}d.\ \frac{8}{100}$, which we multiply by 2, for the two years.

Exercises.—What is the interest on the following sums?

1.	£847	16	8	for	2	years, at	3	per cent.	Ans.	£50	17	$4\frac{3}{4}$	$\frac{1}{5}$
2.	1256	10	$6\frac{1}{2}$	"	7	"	$2\frac{1}{2}$	"	"	219	17	$10\frac{0}{4}$	$\frac{11}{20}$
3.	732	15	$9\frac{3}{4}$	"	5	"	$2\frac{1}{4}$	"	"	82	8	$9\frac{1}{4}$	$\frac{31}{80}$
4.	179	11	$7\frac{1}{2}$	"	4	"	$2\frac{3}{4}$	"	"	19	15	$0\frac{3}{4}$	$\frac{36}{80}$
5.	273	19	10	"	$3\frac{1}{2}$	"	3	"	"	28	15	$4\frac{1}{2}$	$\frac{9}{25}$
6.	2685	18	$6\frac{1}{4}$	"	$9\frac{1}{4}$	"	$3\frac{1}{4}$	"	"	807	9	$1\frac{1}{2}$	$\frac{409}{1600}$
7.	987	1	$8\frac{1}{4}$	"	$5\frac{3}{4}$	"	$4\frac{1}{2}$	"	"	255	8	$1\frac{3}{4}$	$\frac{607}{800}$
8.	1751	9	$2\frac{1}{2}$	"	11	"	$3\frac{7}{8}$	"	"	746	11	$2\frac{1}{4}$	$\frac{241}{400}$

III. To find the Interest for any number of Months.

Rule.—Find the interest for a year by Rule I. and then take the $\frac{1}{2}$, $\frac{1}{3}$, or some other aliquot part of the amount, according to the number of months.

Example.—What is the interest on £80 for three months, at 5 per cent.?

4) £4
£1 *Answer.*

Here £4, the interest of £80 for 1 year, is divided by 4, as 3 months are $\frac{1}{4}$ of a year.

Interest at 5 per cent. may be conveniently found by reckoning the pounds of the given sum as so many pence, and then multiplying them by the number of months (interest at 5 per cent. being equal to 1*d.* a £ per month).

Thus, to find the interest on £24 for 5 months, reckon the £24 as 24*d.* = 2*s.*, then multiply by 5, and the answer is 10*s.* If there are shillings and pence in the given sum, add the proportion of the interest for £1.

Exercises.—What is the interest on the following sums?

1.	187	16	10	for	5	months, at	$4\frac{1}{2}$	per cent.	Ans.	£3	10	$5\frac{1}{4}$	$\frac{3}{20}$
2.	1395	6	8	"	3	"	$2\frac{1}{2}$	"	"	8	14	5	
3.	1500	0	0	"	9	"	$4\frac{1}{2}$	"	"	50	12	6	
4.	6000	0	0	"	2	"	2	"	"	20	0	0	
5.	1000	0	0	"	1 yr. $3\frac{1}{2}$ mo.		5	"	"	64	11	8	

IV. To find the Interest for Days.

Rule.—Multiply the principal by the number of days, and the product by *twice* the rate per cent.; then divide the result by 73,000: the quotient is the answer required.

Note.—Interest at 5 per cent. is found by multiplying the principal by the number of days, and dividing the product by 7300. This is merely an abridgment of the general rule.

Example.—What is the interest on £235, 10*s.* for 125 days, at 3 per cent.?

```
             £235 10
                125
             -------
          29,437 10
                  6
             -------
73,000)176,625  0( £2 8 4¼ Ans.
```

Here we multiply the principal by the number of days, 125, and the product by twice the rate per cent., 6, and then divide the last product by 73,000 for the answer.

THE DIVISION BY 73,000 may be readily performed by the following rule, termed 'the third, tenth, and tenth rule:'—

RULE.—After multiplying by the number of days, and twice the rate per cent. write below the *pounds* of the product (the shillings and pence not being reckoned), $\frac{1}{3}$ of itself, $\frac{1}{10}$ of the third, and $\frac{1}{10}$ of that tenth; and add the four lines together: then cancel the two last figures; reckon the two next figures as so many farthings—less 1 farthing for every 25; the double of the next figure as so many shillings; and the rest of the figures as so many pounds. The whole forms the answer, and is nearly correct—there only requires a farthing to be subtracted for every £10 in the answer, to give nearly the exact interest.

Example.—Divide £357,200 by 73,000.

	357,200
$\frac{1}{3}$ =	119,066
$\frac{1}{10}$ of $\frac{1}{3}$ =	11,906
$\frac{1}{10}$ " $\frac{1}{10}$ =	1,190
	4893~~62~~
Ans.	£4 19 $10\frac{1}{2}$

Here, after adding 1-3d, 1-10th, and 1-10th to the sum, we cancel the two last figures of the total: we then reckon 93 as so many farthings—less 3, as there are 3 times 25 in 93, making 90, or 1*s.* $10\frac{1}{2}$*d.*; the double of 9 is reckoned as so many shillings, or 18*s.*, and the other figure, 4, as 4 pounds. The answer is £4, 19*s.* $10\frac{1}{2}$*d.* which is about $\frac{1}{4}$ more than the exact sum.

THE DIVISION by 7300, in the case of 5 per cent. is performed in the same way as that by 73,000, only, instead of cancelling the *two* last figures as above, cancel merely the *last* figure.

Exercises.—What is the interest on the following sums?

1.	£5	0	0	for	60	days, at	3	per cent.	Ans.	£0	0	$5\frac{3}{4}$	$\frac{49}{73}$
2.	84	0	0	"	90	"	4	"	"	0	16	$6\frac{3}{4}$	$\frac{129}{365}$
3.	781	0	0	"	47	"	5	"	"	5	0	$6\frac{3}{4}$	$\frac{81}{365}$
4.	641	12	6	"	36	"	$4\frac{1}{2}$	"	"	2	16	$11\frac{1}{4}$	$\frac{1551}{1825}$
5.	7138	18	4	"	158	"	$3\frac{1}{4}$	"	"	100	8	$8\frac{0}{4}$	$\frac{818}{1825}$
6.	1127	0	0	"	213	"	5	"	"	1159	17	$8\frac{0}{4}$	$\frac{128}{365}$

7. What will £328, 16*s.* 11*d.* amount to, at $2\frac{5}{8}$ per cent. interest, from January 7 to March 29?* . Ans. £330, 15*s.* $2\frac{3}{4}$*d.* $\frac{1023}{73000}$

8. Find the interest on £584, 11*s.* $3\frac{1}{2}$*d.* from February 21 to August 17, at $3\frac{3}{4}$ per cent.? . . Ans. £10, 12*s.* $7\frac{1}{4}$*d.* $\frac{821}{14600}$

9. What will £816, 17*s.* $6\frac{3}{4}$*d.* amount to, at $4\frac{3}{8}$ per cent. interest, from June 14 to September 25? . Ans. £826, 19*s.* $3\frac{0}{4}$*d.* $\frac{39963}{58400}$

10. What is the interest of £875, 14*s.* 6*d.* from August 10 to December 14, at $4\frac{1}{4}$ per cent.? . . Ans. £12, 16*s.* $11\frac{1}{2}$*d.* $\frac{427}{9125}$

11. What is the interest of £697, 8*s.* $5\frac{1}{4}$*d.* from June 17 to December 31, at 5 per cent.? . . Ans. £18, 16*s.* $5\frac{0}{4}$*d.* $\frac{1}{292}$

* NOTE.—In exercises 7 to 11, we require to calculate, in each case, the number of days from the one given period to the other. The following is the method of calculating the number of days—as, for instance, from January 7 to March 29—

Jan.	24
Feb.	28
Mar.	29
Ans.	81 days.

Here we take the remaining days in January *after* the 7th, then the days in February, and the days in March *up to* the 29th, and add them all together. The day reckoned *from* is not counted, but that *to* which we reckon, is counted.

V. To calculate the Interest on sums or debts when partial payments are made.

A partial payment of a debt is made, when part of the principal is repaid after a certain time, leaving the balance at interest for a longer period.

RULE.—Multiply each sum or balance due, by the number of days that it lies at interest, and add together the different products; then multiply the sum-total by twice the rate per cent. and divide the result by 73,000.

Example.—A sum of £300 was borrowed on March 16; of which £50 were repaid on April 7, £100 on July 16, and the balance, including interest at 4 per cent., on October 11; how much will the last payment amount to?

		£		Days at interest.		Products.
Mar. 16.	Principal,	300	×	22	=	6,600
April 7.	Paid,	50				
	Balance,	250	×	100	=	25,000
July 16.	Paid,	100				
	Balance,	150	×	87	=	13,050
						44,650
						8
				73,000	)	357,200
	Interest due at Oct. 11,					£4 17 10¼
	Balance at July 16,					150 0 0
					Ans.	£154 17 10¼

Here £300 lies at interest from March 16 to April 7, that is, 22 days; we therefore multiply 300 by 22. On April 7, £50 were paid, and the balance, £250, lies at interest from April 7 to July 16, that is, 100 days; we therefore multiply 250 by 100. Again, on July 16, £100 were paid, and the balance, £150, lies at interest from July 16 to October 11, that is, 87 days; hence we multiply 150 by 87. We now add the products, and multiply their sum, 44650, by 8, twice the rate per cent. and then divide the product by 73,000; the interest is £4, 17*s.* 10¼*d.* to which we add £150, the balance due at July 16; and the sum, £154, 17*s.* 10¼*d.* is the amount of the last payment.

Exercises.

1. A person borrowed £500 on February 2: he repaid ¼ of this sum on May 15, ¼ on August 1, ¼ on November 11, and the remaining ¼, together with the interest, on December 31. Required the amount of the last payment, . . Ans. £140, 6*s.* 10¾*d.* $\frac{66}{73}$

2. Required the interest at 3½ per cent. on £320, due on September 11, 1846, of which £116 were paid on November 23, £50 on December 21, £100 on January 19, 1847, £30 on February 22, and the balance on May 11, . . Ans. £3, 16*s.* 6¼*d.* $\frac{211}{365}$

3. I lent £456 to a friend on March 14, and received as part-payment, £66 on April 30, £130 on July 11, £120 on August 15, £100 on October 19, and the balance on November 30; how much interest have I to receive, at this last date, at 3½ per cent.?
Ans. £6, 13*s.* 0¾*d.* $\frac{1821}{1825}$

VI. To calculate the Interest on Accounts-current.

An Account-current is an account in which is drawn out, in Dr. and Cr. (Debtor and Creditor) columns, a statement of the transactions that have taken place between two parties.

Example.—Required the interest at 4 per cent. on the following account-current to June 30.

Dr. Mr J. Simpson, in Account-current with R. Duff. *Cr.*

			£	s.	d.				£	s.	d.
1854.						1854.					
Jan.	18	To Goods,	360	14	9	Jan.	1	By Goods,	102	17	0
Mar.	23	" do.	468	11	6	"	30	" Cash,	400	0	0
Apr.	17	" do.	124	10	0	Apr.	24	" Bill,	350	0	0
May	22	" do.	739	15	4	June	16	" Goods,	690	14	4
June	30	" Interest	3	8	2¼	"	30	" Balance forwd.	153	8	5¼
			1696	19	9½				1696	19	9½
June	30	To Balance,	153	8	5¼						

This account is drawn out by R. Duff, and sent to J. Simpson on June 30. On the left or Dr. side are written all the sums that Simpson owes to Duff; and on the Cr. side the sums that Duff owes to Simpson. Interest at 4 per cent. is then calculated on the account, as below; and as Duff finds that Simpson is due him £3, 8*s.* 2¼*d.* of interest, he enters it on the Dr. side. He next adds the Dr. side, and finds the amount to be £1696, 19*s.* 9½*d.*; then the Cr. side, which amounts to £1543, 11*s.* 4*d.* The difference between these, £153, 8*s.* 5¼*d.* is entered on the Cr. side, to balance the account, and then transferred to the Dr. side of a new account, shewing that J. Simpson is owing R. Duff £153, 8*s.* 5¼*d.*

Interest on the account is calculated as follows:—

					Days.		Dr.	
Jan. 18,	To £ 360	14	9	×	163	. . . =	58800	
Mar. 23,	" 468	11	6	×	99	. . . =	46389	
Apr. 17,	" 124	10	0	×	74	. . . =	9213	
May 22,	" 739	15	4	×	39	. . . =	28851	146,253
							Cr.	
Jan. 1,	By £ 102	17	0	×	181	. . . =	18616	
" 30,	" 400	0	0	×	151	. . . =	60400	
Apr. 24,	" 350	0	0	×	67	. . . =	23450	
June 16,	" 690	14	4	×	14	. . . =	9670	112,136

31,117
8
73,000) 248,936
Interest, £3 8 2¼

As the account is supposed to be settled up to June 30, the interest is calculated according to Rule V. page 100, on all the sums on both sides of the account, up to that date: as, for instance, on £360, 14*s.* 9*d.* from January 18 to June 30, or for 163 days; and so on. The products of the Dr. side are placed in one column, and of the Cr. side in another; each column is then added, and the smaller of the two sums deducted from the greater; the interest is then calculated on the difference; and the Dr. products in this case being the greater, the interest is entered on the Dr. side of the account-current.

In multiplying the sums by the number of days, the shillings and pence of the products have, for convenience, been left out.

Exercises.—1. What is the interest on the following account to December 31, at $4\frac{1}{2}$ per cent.?

Dr. Mr J. Miller in Account-current with W. Ferrie. *Cr.*

July 17, To Goods, . .	£450	July 30, By Cash, . .	£340
Aug. 15, 〃 Cash, . . .	300	Sept. 17, 〃 do. . .	693
Sept. 4, 〃 do. . .	721	Oct. 15, 〃 do. . .	960
Nov. 11, 〃 do. . . .	875	Nov. 29, 〃 do. . .	123

Ans. Interest due to W. Ferrie, £5, 4*s.* $6\frac{1}{4}$*d.* $\frac{1727}{1825}$

2. Allan & Son are owing A. Jones £452 on July 5: they grant him a bill for £165, payable on July 13, and another for £225, payable on Aug. 1; they are due him for goods £347 on Aug. 25, and £127 on Sept. 11; they grant him a bill for £439, due on Oct. 10; on Nov. 3, they send goods to the value of £716; on Dec. 17, they receive from him £560. State the account current sent to Jones on Dec. 31, allowing interest at 5 per cent. Ans. Allan & Son owe Jones for interest, £1, 0*s.* $0\frac{1}{2}$*d.* $\frac{46}{73}$; Jones is owing them, £57, 19*s.* $11\frac{1}{4}$*d.* $\frac{27}{73}$

Explanation of the Rules.—The Rules for finding interest are virtually the same as those of Simple and Compound Proportion, according to the nature of the case.

Interest for *one* year is a case of Simple Proportion: for instance, the question, 'What is the interest on £40 for one year at 5 per cent.?' may be stated thus:—

$$\begin{array}{ccccc} £ & & £ & & £ \\ 100 & : & 40 & :: & 5 \end{array}$$

The question, as a case of Simple Proportion, may be expressed in this way—If the interest on £100 is £5, what will be the interest on £40?

Interest for more than one year, or for days, is a case of Compound Proportion: for instance, the question, 'What is the interest on £250 for 45 days, at 4 per cent.?' may be stated thus:—

$$\begin{array}{ccccc} £ & & £ & & £ \\ 100 & : & 250 & :: & 3 \\ 365 \text{ days.} & & 45 \text{ days.} & & \\ \hline 36{,}500 & & 11{,}250 & & \\ \hline 73{,}000 & & & & \end{array}$$

The meaning is:—If the interest of £100 for 365 days is £3, what will be the interest of £250 at the same rate for 45 days? hence it is a question in Compound Proportion. For convenience in working, the first term is doubled, making 73,000; and to put the second term on an equality, it would require to be also doubled, but it is more convenient, and has the same result, to double the third term, and thus multiply by twice the rate per cent.

COMPOUND INTEREST is computed by adding to the principal the interest due at any given time, as—at the end of a year; then reckoning interest on this new amount for a similar period, and again adding it as before; and so on.

Example.—What will £100 amount to in 3 years, at 5 *per ct.* compound interest?

£100	0	0	
5	0	0	
105	0	0	1st year.
5	5	0	
110	5	0	2d year.
5	10	6	
Ans. £115	15	6	3d year.

Here we add to the principal, the interest for one year, £5; then to the amount of the two sums, the interest for the second year, £5, 5*s.*; and to the last amount, the interest for the third year, £5, 10*s.* 6*d.* The total amount at the end of the third year is £115, 15*s.* 6*d.*—namely, principal, £100; and compound interest, £15, 15*s.* 6*d.*

Compound Interest may be calculated in this way when the time is only two or three years; but for longer dates, this would be a tedious process, and another *method is employed*, which is treated of in the Advanced work on Arithmetic.

DISCOUNT.

DISCOUNT is a charge of so much per cent. made by bankers and others, for advancing money upon bills, &c. before they are due. Discount is *deducted* from the given sum, and is thus the reverse of interest.

For an account of Bills, and the discounting of them, see page 134.

Discount is also the term applied to the allowance or deduction frequently made at the settlement of accounts. Thus, a person who is owing an account of £100, on settling it, may receive an allowance of 2½ per cent.; he would therefore pay only £97, 10*s*. the remaining £2, 10*s*. being allowed as discount.

When discount at so much per cent. is stated without any time being specified, as, 'discount 10 per cent. on £250,' the meaning is, that discount is to be reckoned at the rate of £10 for every £100 in the sum.

DISCOUNT is calculated in the same way as interest, whether for years, months, or days. When no particular time is specified, it is calculated by Rule I. of Interest.

NOTE.—Discount at 10 per cent. is calculated by merely taking $\frac{1}{10}$th of the given sum—that is, dividing it by 10: thus, discount on £370 at 10 per cent. is £37.

Example.—What is the discount and nett proceeds of a bill for £250, dated Aug. 1, due at 4 months after date, which was discounted on Sept. 23, at 4 per cent.?

	£	*s.*	*d.*
Bill,	250	0	0
Deduct discount for 72 days,	1	19	6
Nett proceeds,	£248	0	6

Here the bill is payable on December 4, reckoning the *three days of grace* (see page 135), that is in 72 days after September 23, the day on which it was discounted. The discount for 72 days is calculated as in Interest, Rule IV. and amounts to £1, 19*s*. 6*d*. which being deducted from the bill, leaves £248, 0*s*. 6*d*. as the nett proceeds.

In discounting bills, any farthings in the answer are considered, by bankers, as a penny; thus, if the discount amounts to £3, 2*s*. 2¼*d*. it is reckoned as £3, 2*s*. 3*d*.

Exercises.

What is the discount on the following sums?

1.	£124	7	6	at	2½	per cent.	. . .	Ans.	£3	2	2¼
2.	219	12	4	〃	4	〃	. . .	〃	8	15	8¼
3.	385	5	10	〃	5	〃	. . . .	〃	19	5	3½
4.	572	9	8	〃	7½	〃	. . .	〃	42	18	8½
5.	621	18	2	〃	10	〃	. . . .	〃	62	3	9¾

6. A Bill dated Jan. 1, at 3 months' date, for £739, 16*s*. 11*d*. was discounted on Feb. 14; what was the discount and nett proceeds?

Ans. Discount, £4, 19*s*. 3¾*d*.; Nett proceeds, £734, 17*s*. 7*d*.

7. Required the discount and the nett proceeds on the following bills, at 4 per cent. which were discounted on April 4: one for £174, dated February 24, at 4 months; one for £1000, dated March 15, at 2 months,

Ans. Discount, £6, 8*s.* 5½*d.*; Nett proceeds, £1167, 11*s.* 6¼*d.*

8. The following bills were discounted on June 27:—No. 20, for £360, dated April 14, at 5 months; No. 23, for £721, dated May 2, at 3 months; No. 31, for £875, 10*s.* dated May 15, at 2 months; and No. 32, for £691, 15*s.*, dated June 3, at 4 months. What was the nett proceeds, allowing interest at 4 per cent. and commission at ½ per cent.? . . . . . . Ans. £2619, 0*s.* 4¾*d.*

COMMISSION AND BROKERAGE.

COMMISSION is a charge of so much per cent. made by an agent for buying or selling goods, &c. on account of another. The rate varies from 1 to 10 per cent.

BROKERAGE is a similar charge made by persons termed brokers, for assisting others in buying or selling goods, shares, &c. The rate is usually less than 1 per cent.

COMMISSION and BROKERAGE are calculated by multiplying the given sum by the rate per cent. and dividing the product by 100, as in Interest, Rule I.

When the rate is 1, 2, 3 per cent. &c. *pounds* are meant: when the rate is expressed in shillings and pence, take proportionate aliquot parts; thus, for 15*s.* take ¾ of £1, or for 3*s.* 4*d.* take ⅙ of £1.

Example.—What is the commission on £700, 10*s.* at 4 per cent.?

	£	s.	d.
	£700	10	0
			4
100)	2802	0	0
Ans.	£28	0	$4\frac{3}{4}\ \frac{1}{5}$

Here we multiply by 4, the rate per cent. and then divide by 100.

Exercises.

What is the commission on the following sums?

1.	£325	19	11	at 5 per cent. . . .	Ans.	£16	5	$11\frac{3}{4}$	$\frac{4}{5}$
2.	695	10	$11\frac{1}{2}$	〃 3 〃 . . .	〃	20	17	$3\frac{3}{4}$	$\frac{39}{50}$
3.	384	17	9	〃 2 〃 . . .	〃	7	13	$11\frac{1}{4}$	$\frac{21}{25}$
4.	1234	15	$6\frac{3}{4}$	〃 $4\frac{1}{2}$ 〃 . . .	〃	55	11	$3\frac{1}{2}$	$\frac{83}{200}$

What is the brokerage on the following sums?

1.	£439	12	6	at 3*s.* 4*d.* per cent. . .	Ans.	£0	14	$7\frac{3}{4}$	$\frac{2}{5}$
2.	975	10	3	〃 5*s.* 6*d.* 〃 . . .	〃	2	13	$7\frac{3}{4}$	$\frac{363}{1000}$
3.	1025	15	4	〃 $\frac{1}{8}$ 〃 . . .	〃	1	5	$7\frac{1}{2}$	$\frac{23}{25}$
4.	731	17	$9\frac{1}{2}$	〃 $\frac{4}{5}$ 〃 . . .	〃	5	17	$1\frac{0}{4}$	$\frac{114}{125}$

INSURANCE.

INSURANCE is a contract by which certain persons or insurance offices engage to make good to the party insuring, losses he may sustain of ships or their cargoes at sea, or of houses or goods by fire.

The parties who take upon themselves the risk, are called the *insurers*, or *underwriters;* and the person protected, the *insured;* the sum paid to the insurers is called the *premium;* the stamped paper on which the contract is written, the *policy of insurance;* and the stamp-duty on the policy, the *policy-duty*. Besides the premium and duty, there is, in some cases, a commission charged.

Sums of money are also insured on persons' lives; an individual contracting to pay a certain premium annually during his life, has a sum insured to be paid to his family at his decease.

I. To find the Premium on the Sum insured.

RULE.—Multiply the given sum by the rate per cent. and divide the product by 100, as in Commission.

When the rate is 1, 2, 3 per cent. &c. *pounds* are meant: when the rate is expressed in shillings and pence, take proportionate aliquot parts; thus, for 15*s*. take $\frac{3}{4}$ of £1, or for 3*s*. 4*d*. take $\frac{1}{6}$ of £1.

When the rate is expressed in guineas, calculate as if it were in pounds, and to the result add $\frac{1}{20}$, for the premium required.

Example.—What is the expense of insuring a cargo valued at £648; the premium being 35*s*. or $1\frac{3}{4}$ per cent.?

$$\begin{array}{r} £648 \\ 1\frac{3}{4} \\ \hline 648 \\ \frac{1}{2} = 324 \\ \frac{1}{4} = 162 \\ \hline 100\,)\,1134\,(\,\textit{Ans.}\ £11\quad 6\quad 9\frac{1}{2}\ \frac{2}{5} \end{array}$$

Here we multiply £648 by $1\frac{3}{4}$, the rate per cent. and then divide the product by 100.

Exercises.—What is the premium on the following sums?

1.	£376	12	6	at $1\frac{1}{2}$ per cent. . .	Ans.	£5	12	$11\frac{3}{4}$	$\frac{2}{5}$
2.	742	6	8	" 36*s*. " .	"	13	7	$2\frac{3}{4}$	$\frac{13}{25}$
3.	780	0	0	" $2\frac{1}{2}$ guineas per cent.	"	20	9	6	
4.	1965	0	0	" $3\frac{1}{4}$ " "	"	67	1	$1\frac{1}{4}$	$\frac{2}{5}$

5. Find the expense of insuring household property to the amount of £650, the premium being 1*s*. 6*d*. per cent. on the sum insured, and the policy-duty 3*s*. per cent.* . Ans. £1, 9*s*. 3*d*.

6. What is the expense of insuring £3587 on a building, the premium being at 2*s*. 6*d*. per cent. on the amount insured, and the policy-duty 3*s*. per cent.? . . . . Ans. £9, 17*s*. $3\frac{1}{4}$*d*. $\frac{17}{25}$

* The policy-duty is always charged on even hundreds; thus, if the sum insured is £650, the duty is charged on £700.

II. To find how much must be insured in order to cover a given Sum, besides paying all expenses of Premium, &c.

A merchant sometimes insures not only the value of his property, but also the premium, duty, commission, and other charges; so that, in case of loss, he may be entitled to receive from the underwriters, or insurance-office, a sum equal to the value of the property and expenses of insurance. In this case the property is said to be *covered*.

Rule.—Subtract the percentage to be paid for premium, duty, and commission, from £100; then state the case as a question in Simple Proportion, thus: 'If the remainder—that is, £100 less the expenses—requires to be insured for £100, in order to be covered, what will £2820 require to be insured for?'

Example.—What sum must be insured to cover £3091 in case of loss, the premium being 51*s.* per cent., policy-duty 4*s.* per cent., and commission ¼ per cent.?

Deduct from . . .		£100
Premium,	51/	
Policy-duty,	4/	
Commission,	5/	3
		£97 : 100 :: 3091 : 3300 *Ans.*

Here, on deducting the premium, &c. on £100, from that sum, the remainder is £97; and, consequently, we must insure for £100, in order to cover £97; the question, therefore, is stated thus: 'If £97 must be insured for £100, what must £3091 be insured for?'

Exercises.

1. What sum must be insured to cover £750, premium 2½ guineas, and commission ¼ per cent.? . Ans. £774, 3*s.* 10¼*d.* $\frac{26}{31}$

2. How much must be insured to cover £675, the premium being 4½ guineas, and commission ½ per cent.? Ans. £710, 4*s.* 11$\frac{0}{4}$*d.* $\frac{882}{7603}$

3. What sum must be insured to cover £2884, 10*s.* in case of loss, the premium being 3 guineas per cent., policy-duty 4*s.* per cent., and commission ½ per cent.? . . . Ans. £3000.

4. How much must be insured to cover £1000, the whole expenses attending the insurance being £8, 7*s.* 6*d.* per cent.? Ans. £1091, 8*s.* 1$\frac{0}{4}$*d.* $\frac{716}{731}$

5. What sum must be insured to cover £1250, the whole expenses attending the insurance being £5, 15*s.* per cent.? Ans. £1826, 5*s.* 2¼*d.* $\frac{307}{377}$

PROFIT AND LOSS.

PROFIT and LOSS refers to calculations of the profits or losses of merchants in buying and selling goods.

The price at which a merchant buys his goods, is termed the *cost* price, and that at which he sells them, the *selling* price.

When they are sold for more than they cost, there is a *profit* on the transaction; and when sold for less, there is a *loss*.

The profit or loss is calculated on the *cost price*, and is usually stated at so much per cent.

The method of working questions of Profit and Loss, will be seen from the following examples:—

I. TO ASCERTAIN THE TOTAL PROFIT OR LOSS IN SELLING A QUANTITY OF GOODS—THE RATES AT WHICH THEY WERE BOUGHT AND SOLD BEING GIVEN.

Example.—A merchant purchased 7 tons of iron rods, at £10, 17*s*. 9*d*. per ton, and sold them again at 16*s*. 4½*d*. per cwt.; how much did he gain on the whole?

Sold 7 tons = 140 cwt. at	£0	16	4½	=	£114	12	6
Bought 7 tons . . . "	10	17	9	=	76	4	3
				Total gain,	£38	8	3

The prices at which the goods were bought and sold are calculated, and the amount of the one is then deducted from that of the other.

II. TO ASCERTAIN THE PROFIT OR LOSS *per cent.*—THE COST AND SELLING PRICE BEING GIVEN.

Example.—If indigo be bought at 3*s*. a lb. and sold at 3*s*. 9*d*. a lb. what is the gain per cent.?

s.		*d.*		£		£
3	:	9	::	100	:	25 *Ans.*

Here the gain upon 3*s*. is 9*d*.; and to find the gain per cent. state the question as in Simple Proportion: 'If 3*s*. gain 9*d*. what will £100 gain?'

NOTE.—This question, and the examples under Rules III. and IV. are stated in the order in which the meaning may be expressed in words, according to the rule in Simple Proportion, given at page 59. The examples under Rule V. are stated according to the rule in Simple Proportion, page 56.

III. To ascertain the *Selling* Price—the cost, and the profit or loss per cent. being given.

Example 1; *where there is a profit.*—I bought a quantity of tea at 2*s.* 6*d.* a lb.; at what price must I sell it per lb. in order to gain 10 per cent.?

£		£		s. d.		s. d.
100	:	110	::	2 6	:	2 9 *Ans.*

Goods costing £100 must, at this rate, be sold for £110, to gain 10 per cent.; and therefore, to find what goods costing 2*s.* 6*d.* must be sold for, to gain the same percentage, state the question thus:—'If £100 sells for £110, what will 2*s.* 6*d.* sell for?'

Example 2; *where there is a loss.*—What is the selling price of a pound of tea, if it has cost me 2*s.* 6*d.* and I have lost 10 per cent. in selling it?

£		£		s. d.		s. d.
100	:	90	::	2 6	:	2 3 *Ans.*

Goods costing £100 must, at this rate, be sold for £90, to lose 10 per cent.; and therefore, to find what goods costing 2*s.* 6*d.* must be sold for, to lose the same percentage, state the question thus:—'If £100 is sold for £90, what will 2*s.* 6*d.* be sold for?'

IV. To ascertain the *Cost* Price—the selling price, and the profit or loss per cent. on the cost being given.

Example 1; *where there is a profit.*—What is the cost price of a yard of cloth, which I sold for 16*s.* and thereby gained 10 per cent. on the cost?

£		£		s. d.		s.
110	:	100	::	16 6	:	15 *Ans.*

Goods sold for £110 will, at this rate, cost £100; therefore, to find what goods sold for 16*s.* 6*d.* will cost, state the question thus:—'If £110 cost £100, what will 16*s.* 6*d.* cost?'

Example 2; *where there is a loss.*—What is the cost price of a yard of cloth, which I sold for 18*s.* and thereby lost at the rate of 10 per cent. on the cost?

£		£		s.		£
90	:	100	::	18	:	1 *Ans.*

Goods sold for £90 will, at this rate, cost £100; therefore, to find what goods sold for 18*s.* cost, state the question thus:—'If £90 cost £100, what will 18*s.* cost?'

Note.—In questions of profit and loss, it must be remembered that the calculations are made on the *cost* price, and not on the selling price of the goods.

V. To ascertain what will be the Profit or Loss per cent. at a certain Selling Price—the profit or loss per cent. at another selling price being given.

Example 1; *where there is a profit.*—What will be the percentage gained by selling sugar at £44 a ton, if 5 per cent. is gained by selling it at £42 a ton?

£		£		£		£	
42	:	44	::	105	:	110	
Deduct, as the cost of £110,						100	
The remainder is the percentage required,						£10	viz. 10 per cent. gain.

Here the selling price, £42, which includes a gain of 5 per cent. on the cost, bears the same proportion to £44, the other selling price, that £105—namely, £100 of cost and 5 per cent. added—bears to £100 of cost, with the required percentage added; therefore, state the question thus:—'As £42 is to £44, so is £105 to £100 with the required percentage added.' The answer is £110, from which the cost, £100, is deducted, leaving £10 the required percentage.

Example 2; *where there is a loss.*—What will be the percentage lost by selling sugar at £38 a ton, if 10 per cent. is gained by selling it at £44 a ton?

£		£		£		£	
44	:	38	::	110	:	95	
The cost price of £95 is						100	
The selling price deducted from the cost gives the required percentage . .						£5	viz. 5 per cent. lost.

Here the selling price, £44, which includes a gain of 10 per cent. bears the same proportion to £38, the other selling price, that £110—namely, £100 of cost, and 10 per cent. of gain *added*—will bear to £100, with the required loss per cent. *deducted*; therefore, state the question thus:—'As £44 is to £38, so is £110 to £100, with the required percentage deducted.' The answer is £95, which is deducted from £100, the cost price, and the remainder is the percentage required.

Exercises.

1. A person bought 137 cwts. of pearl sago, at £1, 14*s.* 9½*d.* per cwt. and sold it again at 4¼*d.* per lb.; whether did he gain or lose, and how much on the whole? . Ans. £33, 7*s.* 10½*d.* gain.

2. How much is gained per cent. by selling Muscatel raisins at 91*s.* 3*d.* per cwt. which were purchased at 77*s.* 9*d.*? Ans. $17\frac{113}{311}$

3. A bookseller bought a copy of an Encyclopædia for £22; at what price must he sell it, to gain 20 per cent.? Ans. £26, 8*s.*

4. I have bought cloth at 15*s.* a yard, and lost 5 per cent. in selling it; what was it sold for? . . . . Ans. 14*s.* 3*d.*

5. A merchant sold a chest of tea containing 84 lbs. for £25, 7*s.* and gained at the rate of 20 per cent.; what was the cost per lb.? . . . . Ans. 5*s.* 0¼*d.* $\frac{3}{7}$

6. If 15¼ per cent. be lost by selling Stockholm tar at 15*s.* 4*d.* per barrel, what was the cost? . . . Ans. 18*s.* $1\frac{0}{4}$*d.* $\frac{148}{339}$

7. If 15 per cent. be gained by selling hemp at £32, 15*s.* per ton, what is gained or lost per cent. by selling it at £30, 15*s.* 6*d.*? Ans. £8, 1*s.* 3¼*d.* $\frac{38}{131}$ gain.

8. If 8 per cent. be gained by selling 736 yards of linen for £125, 10*s.*; at what rate must the yard be sold so as to gain 16 per cent.? . . . . Ans. 3*s.* $7\frac{3}{4}$*d.* $\frac{179}{207}$

9. By selling molasses at £1, 10*s.* per cwt. I gained 14 per cent. but the market falling, I was obliged to dispose of the remainder at £1, 2*s.* 6*d.* per cwt.; what was the gain or loss per cent. at this latter price? . . . . Ans. £14, 10*s.* loss.

SHARES, STOCKS.

Shares and Stocks are terms applied to the capital of Joint-stock Companies, and to those large sums of money borrowed by Government, termed the National Debt.

The capital of joint-stock associations is raised among the partners, by shares fixed at a specified sum; the shares may be £10 each, £50 each, or any other sum; and some persons may hold ten shares, while others have fifty; and so on.

The original sum fixed upon for a share is called *par;* thus, if the shares be £10 each, then £10 is par. If they rise in value, a £10 share may become worth £12; in which case it is said to be £2 *above* par; or it may fall in value, and be worth only £9, in which case it is said to be £1 *below* par.

Government Stocks are usually called the *funds:* they take their special designations from the rate of interest paid on them; thus, the 3 per cents. mean that stock on which an annual dividend is paid of £3 per cent.; and so on. The usual practice in buying and selling these, is to offer shares nominally of £100 par, at from £80 to £90, or upwards, each; and the market-price of these fluctuates according to the abundance or scarcity of money, and other circumstances.

The persons who negotiate the sale and purchase of stocks are termed *brokers,* and the charge they make for their trouble is called *brokerage,* or *commission.*

Questions as to stocks, &c. are wrought by Simple Proportion.

Example.—A person invested a certain sum in the 3 per cents. when they were selling at £$94\frac{7}{8}$; what rate per cent. had he for his money?

£		£		£		£	
$94\frac{7}{8}$	:	100	::	3	:	$3\frac{41}{253}$	*Ans.*

Here, by paying £$94\frac{7}{8}$, he receives a yearly sum of £3—that is, £3 is the interest of £$94\frac{7}{8}$—therefore state the question as in Simple Proportion, thus: 'If £$94\frac{7}{8}$ yields £3 of interest, what will £100 yield?'

Note.—In calculating the value of stocks, the sum paid for brokerage is added to the value of stock when bought, but deducted from it when sold. The brokerage in the following exercises is supposed to be at the rate of 1-8th per cent.

Exercises.

1. How much would a person pay for £1500 in the 3 per cents. when the selling price is $91\frac{1}{8}$ per cent.? . . Ans. £1368, 15*s.*

2. A gentleman sold £2100 of the $3\frac{1}{4}$ per cents. at $96\frac{1}{4}$ per cent.; required the nett proceeds, . . . Ans. £2018, 12*s.* 6*d.*

3. What rate of interest does a person obtain by purchasing shares in a railway at £72, the annual dividend being 7 per cent. and the original value of a share £50? . . Ans. $4\frac{31}{36}$ per cent.

PARTNERSHIP.

Partnership is the rule for ascertaining the share of profit or loss belonging to each partner of a company, in proportion to his share of the joint capital.

Simple Partnership is that in which each partner employs his capital for the same period of time.

Compound Partnership is that in which each partner employs his capital for different periods of time.

I. Simple Partnership.

Rule.—Add together the different shares, and state and work the question as in Simple Proportion, for each partner, thus—'If the whole capital gain so much (namely, the total profit), what will each partner's capital gain?'

Example.—Three partners, A, B, C, invested in business £300, £500, and £1200 respectively, and their total profit in a year was £400; what is each partner's share of the profit?

A.	£300						
B.	500						
C.	1200						
	£2000	: £400 ::	£300	:	£60	A's share.
			500	:	100	B's "
			1200	:	240	C's "
					£400	Total profit.

Here we add together the capital of all the partners, and then, to find A's share of profit, state the question thus: 'If the whole capital, £2000, gain £400, what will £300, A's share, gain?' It is then calculated as in Simple Proportion. B and C's shares are found in the same way.

Note.—The question is stated according to the rule in Simple Proportion, given at page 59.

Exercises.

1. Three merchants, A, B, and C, form a joint-capital, of which A contributes £700; B, £350; and C, £1000. At the end of a year their gain is found to be £500. What is each partner's share of the profit? Ans. A's share, £170, 14*s.* $7\frac{1}{2}$*d.* $\frac{18}{41}$; B's, £85, 7*s.* $3\frac{3}{4}$*d.* $\frac{9}{41}$; C's, £243, 18*s.* $0\frac{1}{2}$*d.* $\frac{14}{41}$

2. A, B, C, and D purchase a ship: A pays for 6 shares; B for 5; C for 3; and D for 4. They receive of nett freight for a voyage to Jamaica, £364, 17*s.* 6*d.* How much of this sum ought each to receive? Ans. A, £121, 12*s.* 6*d.*; B, £101, 7*s.* 1*d.*; C, £60, 16*s.* 3*d.*; D, £81, 1*s.* 8*d.*

3. Three merchants, L, M, and N, continue in trade for a year, with a joint-stock of £3500. At the end of that time, L's share of the gain was £125; M's, £240; and N's, £135. What was each partner's stock?

Ans. L's stock, £875; M's, £1680; N's, £945.

4. A, B, and C enter into partnership for a year, with a joint-stock of £8900: A contributes £4000; B, £2140; and C the remainder. At the end of the year, their gain is found to be £1483, 6*s.* 8*d.* C managed the business, and was to have a salary of £445 for his trouble. What portion of the gain belongs to each partner? Ans. A, £466, 13*s.* 4*d.*; B, £249, 13*s.* 4*d.*; C, £767.

II. Compound Partnership.

Rule.—Multiply each share by the time it has been employed in the business, and add together the products: then state and work the question for each partner, as in Simple Partnership; only use the *products* of the sums instead of the sums themselves.

Example.—Three partners, A, B, C, invested the following sums in business:—A, £400, for 6 months; B, £600, for 9 months; C, £1000, for 12 months; and they gained £300: what is each partner's share of the profits?

		Months.								
A.	£400 ×	6	£2,400							
B.	600 ×	9	5,400							
C.	1000 ×	12	12,000	£	£					
	Total products,		£19,800	: 300 ::	2,400 :	£36	7	$3\frac{1}{4}$	$\frac{1}{11}$	A's share.
					5,400 :	81	16	$4\frac{1}{4}$	$\frac{5}{11}$	B's 〃
					12,000 :	181	16	$4\frac{1}{4}$	$\frac{5}{11}$	C's 〃
						£300	0	0		Total profit.

Here each share is multiplied by the time it is employed, and the products are then added together: in order to find A's share, we say, 'If £19,800, the total products, gain £300, what will £2400, the product of A's share, gain?' It is then calculated as in Simple Proportion. B and C's shares are found in the same way.

Exercises.

1. A's stock of £340 was 4 months in trade; B's, of £510, was eight months; and C's, of £850, was 10 months; they gain £270, 13*s.* 6*d.*: what was each partner's share of the gain?
Ans. A's, £26, 8*s.* $1\frac{3}{4}$*d.* $\frac{1}{41}$; B's, £79, 4*s.* $5\frac{1}{4}$*d.* $\frac{3}{41}$;
C's, £165, 0*s.* $10\frac{3}{4}$*d.* $\frac{37}{41}$

2. Two merchants, A and B, entered into partnership for 2 years: A contributed to the capital £960, and B £1500. After 8 months of the time had elapsed, they admit C, with a capital of £720. On balancing their books at the end of the period, they found that their nett gain amounted to £847, 15*s.* How must this gain be divided among them? Ans. A's, £276, 16*s.* $3\frac{3}{4}$*d.* $\frac{33}{49}$;
B's, £432, 10*s.* $6\frac{2}{4}$*d.* $\frac{24}{49}$; C's, £138, 8*s.* $1\frac{3}{4}$*d.* $\frac{41}{49}$

3. A, B, and C enter into partnership for 2 years: A put in at first £700, and after 8 months £250 more; B put in £650, and after 15 months he took out £300; C put in £850, and after 10 months £400 more, but at the end of 18 months he withdrew £900. During their copartnership the gains amounted to £1684, 12*s.*; *what was each* man's share? Ans. A, £645, 5*s.* $11\frac{1}{2}$*d.* $\frac{10}{181}$;
B, £400, 4*s.* $2\frac{2}{4}$*d.* $\frac{88}{181}$; C, £639, 1*s.* $10\frac{1}{4}$*d.* $\frac{93}{181}$

EQUATION OF PAYMENTS.

EQUATION is the rule for ascertaining the time at which two or more sums payable at different dates, by one person to another, may be paid at *one equivalent* date, without loss to either party.

RULE.—1. Write the different sums or debts below one another, and multiply each of them by the time that has to elapse before it is due, placing the products opposite each sum.

2. Add the debts in one sum for a divisor, and their products in another sum for a dividend: then divide the one by the other—that is, the sum of the products by the sum of the debts—and the quotient is the equated or average time required for paying the whole at once.

Example.—A gentleman owes £60, payable in 72 days; £85, in 128 days; £70, in 176 days; and £105, in 320 days. Required the average time at which the whole ought to be paid.

£		Days.		Products.	
60	×	72	=	4320	
85	×	128	=	10880	
70	×	176	=	12320	
105	×	320	=	33600	
320			320)	61120	191 days. *Ans.*

Here we multiply £60 by 72, the number of days before it is due; £85 by 128; £70 by 176; and £105 by 320. We then divide 61120, the sum of the products, by 320, the amount of the debts; and the quotient, 191, is the average number of days.

Exercises.

1. A gentleman owes £56, payable in 40 days; £72, in 108 days; £106, in 175 days; £230, in 241 days; and £960, in 342 days. Required the average time at which the whole ought to be paid, Ans. $289\frac{195}{356}$ days.

2. If a person owe £100, payable in 2 months, and £750, payable in 7 months; what is the just time for the payment of the two debts? . . . . Ans. $6\frac{7}{17}$ months.

3. What is the equated time for the payment of four debts—the first, £250, due in one year; the second, £560, payable in $1\frac{1}{2}$ years; the third, £490, due in 2 years; and the fourth, £1000, due in $3\frac{1}{2}$ years? . . . . Ans. $2\frac{97}{230}$ years.

4. A person has to pay £1750 as follows:—£300 in 4 months; £125 in 5 months; £365 in 8 months; £400 in 10 months; and the rest in a year. What is the equated time for the payment of the whole? . . . . Ans. $8\frac{293}{350}$ months.

AVERAGE.

An average number is one that is intermediate between several other given numbers. Thus, if there are 4 numbers, 5, 6, 9, 8, their average is 7, because 4 numbers, each of which is 7, will amount to the same sum, namely, 28, as the four given numbers.

I. To Find the Average of several given Quantities.

Rule.—Add together the different quantities, and divide their amount by the *number* of the quantities; thus, if there are 3 different quantities, divide their amount by 3; and the quotient is the average.

Example.—What is the average of 8, 36, 14, 9, 43?

```
           8
          36
          14
           9
          43
       5 )110
Average,  22
```

Here the different quantities are added together, and their sum is divided by 5, because there are 5 different quantities.

II. To find the Average price of Goods, &c. when there are different quantities and different prices.

Rule.—Multiply each quantity by its price; then add the quantities in one sum and the products in another, and divide the sum of the products by the sum of the quantities: the quotient is the average.

Example.—I have bought two yards of cloth, at 10*s.* each; 3 yards at 15*s.*; and 5 at 12*s.*; what is the average price?

```
 2 × 10s. = £1  0  0
 3 × 15s. =   2  5  0
 5 × 12s. =   3  0  0
10         10 )6  5  0
   Average,      12  6
```

Here each quantity is multiplied by its price; and the sum of the products, £6, 5*s.*, is divided by 10, the sum of the quantities.

This rule applies to any other averages in which the quantities and the rates both vary.

Exercises.

1. The revenue of a public trust, during three years, was £44,261, 0*s.* 4*d.*, £47,471, 14*s.* 5*d.*, and £38,006, 5*s.* 11*d.*; what was the average yearly revenue? . Ans. £43,246, 6*s.* 10½*d.* ⅔

2. The temperature, as indicated by the thermometer on the 1st day of November, was 49·10; on the second, it was 40·06; on the 3d, it was 39·00; on the 4th, it was 27·20; on the 5th, 28·; and on the 6th, 21·50; what was the average temperature of the six days? . . . . Ans. 34·14

3. In a class of 6 boys at school, one was aged 14 years and 3 months; the second, 13 years 11 months; the third, 13 years 1 month; the fourth, 12 years; the fifth, 12 years 6 months; and the sixth, 11 years 9 months; what was the average age of the six boys? . . . . Ans. 12 years 11 months.

4. The price of the 4 lb. loaf was in one shop 8*d.*; in a second shop, $7\frac{1}{2}$*d.*; in a third, $7\frac{1}{4}$*d.*; and in a fourth, $6\frac{3}{4}$*d.*; what was the average price of the loaf in the four shops? . . Ans. $7\frac{1}{4}$*d.* $\frac{1}{2}$

5. If a person purchase 12 articles, one of which costs 2*s.*; two, 4*s.* each; two, 5*s.* each; and seven, 8s. each; what did he pay on an average for each article? . . . Ans. 6*s.* 4*d.*

DIVIDEND.

A DIVIDEND is a division among the creditors, of the funds of a debtor, who finds himself unable to pay the debts which he has contracted.

On examining his *assets*—that is, the whole of his property and means—he discovers that he could settle with his creditors, provided each would accept a *dividend* on the amount of his account: this dividend is generally spoken of as at the rate of so much per pound. Supposing the debtor to be owing £1000, and only to possess assets to the value of £250, then he can pay only 5*s.* per pound, or 25 per cent. on his debts; if the creditors are satisfied, the debtor is relieved on making payment to this extent.

THE DIVIDEND is ascertained by dividing the total amount of the assets by the *number* of pounds that form the amount of the debts. As the assets are less than the debts, their amount requires to be converted into shillings or pence, as the case may be, to admit of the division.

DIVIDEND is also the term applied to the profits, at a certain percentage on the amount of the shares, divided among the proprietors of joint-stock companies, &c.

Example.—A person is unable to pay his debts. He owes to A, £440; to B, £160; to C, £224—being in all £824. On examining his affairs, it is found that he possesses property only to the value of £226, 12*s.* What dividend per pound can he pay?

£ £ *s.* 824)226 12(*Ans.* 5*s.* 6*d.* per £1.	Here the assets are divided by 824, the number of pounds forming the debts.

Exercises.

1. What dividend per pound will a person pay who is owing £1000, and whose assets amount to £800? . Ans. 16*s.* per £

2. A person is owing to A, £300; B, £400, 10*s.*; C, £620, 15*s.*; D, £150, 15*s.*; and his whole assets amount to £184, what dividend will he be able to pay? . . . Ans. 2*s.* 6*d.* per £

BARTER.

Barter is the exchanging of one kind of goods for another, in such a way that the value of the goods given away, may be equal to the value of those received.

No general rule can be given for the working of such questions: they must be treated according to the nature of each case. The following will serve as examples:—

Example 1.—A and B barter as follows: A has 1385 yards of linen, at 2*s.* $7\frac{1}{2}$*d.* per yard, for which B gives him £32, 7*s.* 6*d.* ready-money, and for the rest printed calicoes at $10\frac{1}{2}$*d.* per yard. How many yards of calico did A receive?

A gives 1385 yards of linen, at 2*s.* $7\frac{1}{2}$*d.*				=	£181	15	$7\frac{1}{2}$
B gives in money, ...	£32	7	6				
and calicoes at $10\frac{1}{2}$*d.* a yard,	149	8	$1\frac{1}{2}$		181	15	$7\frac{1}{2}$

This sum of £149, 8*s.* $1\frac{1}{2}$*d.* divided by $10\frac{1}{2}$*d.* the price per yard, will give 3415 yards, the number required.

Example 2.—How much coffee, at £7, 9*s.* $6\frac{3}{4}$*d.* per cwt. should I get in exchange for 897 cwts. 1 qr. 14 lbs. of sugar, at $6\frac{3}{4}$*d.* per lb.?

cwts.	*qr.*	*lbs.*		*lbs.*				
897	1	14	=	100506 at $6\frac{3}{4}$*d.*	=	£2826	14	$7\frac{1}{2}$

£7 9 $6\frac{3}{4}$) £2826 14 $7\frac{1}{2}$

Ans. 378 cwts.

Here we convert the given quantity of sugar into lbs. and find the value at $6\frac{3}{4}$*d.* a lb. The amount, £2826, 14*s.* $7\frac{1}{2}$*d.* is then divided by £7, 9*s.* $6\frac{3}{4}$*d.* the price per cwt. of the coffee, and the quotient is the number of cwts. of coffee required.

Exercises.

1. How much tobacco, at £5, 5s. per cwt. must be bartered for 6 cwts. 1 qr. 14 lbs. of snuff, at 4*s.* 6*d.* per lb.?
Ans. 30 cwts. 2 qrs. $11\frac{1}{3}$ lbs.

2. I exchanged 172 yards of black cloth, at £1, 2*s.* 8*d.* per yard, for 688 pair of silk stockings; what was the price of the stockings per pair? . . . . Ans. 5*s.* 8*d.*

3. A delivers to B $31\frac{1}{4}$ yards of cloth, at 5*s.* 6*d.*, and 78 yards of cassimer, at 7*s.* 8*d.*, in barter for wool at 1*s.* 3*d.* per lb.; what quantity of wool does A receive? . . . Ans. $615\frac{8}{10}$ lb.

4. Exchanged 67 cwts. of tobacco, at 8 guineas per cwt. for 600 lbs. of tea, at 7*s.* 4*d.*, and stockings at 2*s.* 8*d.* per pair; how many pair of stockings did I receive? . . Ans. 2571 pair.

INVOLUTION.

INVOLUTION is the term applied to the multiplication of a number one or more times by itself; thus $2 \times 2 = 4$; the result is called a POWER of that number.

The *first* power of a number is the number itself before being multiplied; thus—	2
The *second* power, termed the SQUARE, is the number multiplied by itself; thus—	$2 \times 2 = 4$
The *third* power, termed the CUBE, is the number multiplied by itself, and the product again multiplied by it; thus—	$3 \times 3 \times 3 = 27$

First Power, . .	1	2	3	4	5	6	7	8	9
Second " . .	1	4	9	16	25	36	49	64	81
Third " . .	1	8	27	64	125	216	343	512	729

And so on, with the higher powers.

The *power* of a number is indicated by writing the number with a small figure above it; thus, 3^2 means $3 \times 3 = 9$; 5^3 means $5 \times 5 \times 5 = 125$.

Exercises.

1. What are the squares of 53, 161, 274? Ans. 2809, 25921, 75076
2. " " cubes of 16, 37, 46? " 4096, 50653, 97336
3. If the side of a square table measure 62 inches, how many square inches are contained in the surface of the table? Ans. 3844 sq. inches.
4. The side of a cubic block of granite measures 6 feet; how many solid feet in the block?—and what is its weight, if 1 cubic foot weigh 2654 oz.? Ans. 216 Solid feet; Weight, 319 cwts. 3 qrs. 17 lbs.

EVOLUTION, OR THE EXTRACTION OF ROOTS.

EVOLUTION is the process of finding or extracting the roots of numbers.

The *Root* of any number, is that number which, on being multiplied one or more times by itself, produces the given number.

The *Square* root is that number which, on being multiplied by itself, produces the given number; thus, 4 is the square root of 16, because 4 multiplied by 4 (4×4) produces 16.

The *Cube* root is that number which, on being multiplied by itself, and the product again multiplied by it, produces the given number; thus, 3 is the cube root of 27, because 3 multiplied by 3, and the product again by 3 ($3 \times 3 \times 3$) produces 27.

The sign $\sqrt{}$ (termed the radical sign) placed before a number, indicates that the *square* root of that number is to be extracted; thus $\sqrt{25} = 5$.

The sign $\sqrt[3]{}$ placed before a number, indicates that the *cube* root of that number is to be extracted; thus $\sqrt[3]{729} = 9$.

The extraction of the square and cube roots, serves various useful purposes in connection with the measurement of fields, walls, solid bodies, &c. as will appear from the exercises under the rules for extracting the square and cube roots of numbers.

EXTRACTION OF THE SQUARE ROOT.

RULE.—1. Point off the given number, by means of commas, into periods of two figures each; beginning at the right in integers, and counting towards the *left;* and beginning at the left in decimals, and counting towards the *right:* thus—7,38,69·37,62,1.

If, in pointing off the periods, one figure remain over, either at the left in integers, or right in decimals, it is considered as a period, although consisting of only one figure.

2. Divide the first period by the greatest square root contained in it, and place the quotient at the right of the number, as the first figure of the required root; then annex to the remainder, if any, after the division, the next period of two figures, to form a new dividend.

Example—

2	)5,92,92,25(2435 *Ans.*
2	4
44	)192
4	176
483	)1692
3	1449
4865	)24325

Here the first period, 5, is divided by 2, the greatest square root contained in it; and to the remainder, 1, is annexed the next period, 92, forming the new dividend, 192.

3. Form a new DIVISOR as follows:—Add the previous divisor to itself (in other words, double it), to form *part* of the new divisor; then find how often this is contained in the dividend*—exclusive of its last figure—and annex the quotient† to the partial divisor, which is thereby completed: also place the quotient as the next figure of the root.

Here 2, the previous divisor, is doubled, making 4 for part of a new divisor; and 4 being contained 4 times in 19 (19,2, the dividend, with the last figure left out), 4 is annexed to the partial divisor, thereby forming the *complete* divisor, 44: the quotient, 4, is also placed as the next figure of the root.

4. Multiply the completed divisor by this quotient, and subtract the product from the dividend; then annex to any remainder, the next period of two figures, to form another dividend.

Here the divisor, 44, is multiplied by the quotient, 4, and the product, 176, is subtracted from the dividend, 192: to the remainder, 16, is then annexed the next period, 92, to form another dividend.

5. Form another DIVISOR as follows: —Add to the previous divisor its last figure, to form part of the new divisor; and, as already described in paragraph 3, find how often this is contained in the dividend—exclusive of its last figure—and annex the quotient to the partial divisor, which is thereby completed: also place the quotient as the next figure of the root.

Here, to the previous divisor, 44, is added 4, its last figure, making 48 for part of a new divisor; and 48 being contained 3 times in 169 (169,2, the dividend, with its last figure left out), 3 is annexed to the divisor to complete it, making 483: the 3 is also placed as the next figure of the root.

6. Multiply the completed divisor; and so on, as already described in paragraph 4.

7. Proceed to form new divisors, according to paragraph 5; and go on with the rest of the process according to paragraph 4; till all the periods have been brought down and operated upon; when, if there is no remainder, the *extraction* of the root is completed.

The divisor, 483, is then multiplied by the quotient, 3, and the product, 1449, is subtracted from the dividend, 1692; to the remainder, 243, is annexed the next period, 25, forming a new dividend, 24325; and the process of extracting the next figure of the root is then proceeded with as before. There being now no more periods to bring down, and no remainder, the process is completed.

* *See Note 2, page 119.* † **A less figure sometimes requires to be taken, see Note 3, page 119.**

NOTE 1.—When there is a remainder after all the periods have been brought down, annex a period of two nothings to form a new dividend; and then proceed with the further extraction of the root: the figure of the root thus obtained is a decimal. The process may be carried to any degree of minuteness by annexing more nothings. There are always as many figures in the root as there are periods in the given number; and those figures are decimals in the root, which are extracted from the decimals in the given number.

NOTE 2.—If at any time, on bringing down a new period to form a dividend, the *partial* divisor is found to be greater than the dividend, a nothing [0] must be placed in the root to express this: the next period is then brought down, and annexed to the dividend, and the extraction of a new figure of the root proceeded with as before.

NOTE 3.—It sometimes happens that the completed divisor obtained in the manner described above, proves to be too large, as on multiplying it, it is found to be greater than the dividend: when this is the case, the quotient placed in the root and annexed to the divisor must be reduced as much as is found necessary; thus, if the figure is 9, it must be made 8.

NOTE 4.—The square root of a fraction, is found by extracting the roots of the numerator and denominator.

Exercises.

Find the square root of the following numbers:—

1.	9216, . .	Ans.	96	8.	761·9, .	Ans.	27·602536
2.	27225, .	"	165	9.	5, . .	"	2·236068
3.	119025, .	"	345	10.	17, . .	"	4·1231056
4.	717409, .	"	847	11.	1·7, .	"	1·30384048
5.	62504836, .	"	7906	12.	237·615, .	"	15·4147656
6.	97535376,	"	9876	13.	·0003841,	"	·019598
7.	7619, . .	"	87·286883	14.	$\frac{121}{144}$, .	"	$\frac{11}{12}$

THE MEAN PROPORTIONAL between two numbers—that is, a number that is as many times greater than the one given number, as it is less than the other—is found by multiplying the two given numbers together, and extracting the square root of the product.

15. Find the mean proportional between 16 and 144, Ans. 48

16. " " " " 19 and 41, " 27·91057

17. A cheese, when put into one scale of a false balance, was found to weigh 43 lbs., but when put into the other, it weighed 89 lbs.; what was the true weight of the cheese? . . . Ans. 61·86275 lbs.

THE LENGTH OF THE SIDE OF A SQUARE whose area is given, is found by extracting the square root of the area.

18. The area of a circle is 7085 square inches; what is the side of a square whose area is equal to that of the circle? Ans. 84·1724 inches.

19. A gentleman has three fields: the first measures 2 ac. 1 ro. 30 po.; the second, 3 ac. 2 ro. 15 po.; and the third, 1 ac. 3 ro. 27 po. He wishes another of a square form equal in area to all the three; how many poles must its side measure? . . . . . Ans. 35·665109

20. A person has a field measuring 3 ac. 1 ro. 35 po., which he wishes to exchange for a square one of inferior quality, but $3\frac{1}{2}$ times as large; how many poles in the length of its side? . . . Ans. 44·0738

FOR ADDITIONAL EXERCISES, SEE PAGE 122.

EXTRACTION OF THE CUBE ROOT.

RULE.—1. Point off the given number, by means of commas, into periods of three figures each; beginning at the right in integers, and counting towards the *left;* and beginning at the left in decimals, and counting towards the *right:* thus, 1,784,453·547,46.

If, in pointing off the periods, *one* or two figures remain over, either at the left in integers, or right in decimals, this remainder is considered as a period, although not consisting of three figures.

2. Find the greatest cube root contained in the first period, and place it at the right of the given number, as the first figure of the root: then subtract the *cube* of the root from the first period, and to the remainder, if any, annex the next period of three figures to form a dividend.

Example.—What is the cube root of 178453547?

		178,453,547 (563 *Ans.*
		125
$5^2 \times 300$	= 7500	)53453
$5 \times 6 \times 30$	= 900	
6×6	= 36	
	8436	50616
	36	
	940800	)2837547
$56 \times 3 \times 30$	= 5040	
3×3	= 9	
	945849	2837547

Here, the greatest cube root contained in the first period, 178, is 5, which is placed as the first figure of the required root: the cube of 5*—namely, 125—is then subtracted from the first period, and to the remainder, 53, is annexed the next period, 453, forming the dividend, 53453.

*** $5 \times 5 \times 5 = 125$**

3. Form a DIVISOR for this dividend as follows:—

Multiply the *square* of the root found by 300, for a *partial* divisor.

Find how often this partial divisor is contained in the dividend,* and place the quotient as the next figure of the required root:† then multiply the *previous* figure of the root by this quotient, and the product by 30, placing the result below the partial divisor: also place the *square* of the same quotient (that is, the square of the figure just annexed to the root) below the last product; then add the three sums together, and their amount is the *complete* divisor.

Here, 25, the square of 5, is multiplied by 300, making the partial divisor 7500.

The partial divisor is contained 6 times in the dividend;* the quotient, 6, is therefore placed as the next figure of the required root: the root already found, 5, is then multiplied by the 6, and the product by 30, making 900, which is placed below the partial divisor, 7500. The *square* of the quotient, 6—namely, 36—is placed below the last product, 900. The three sums, 7500, 900, and 36 are then added together, forming the complete divisor, 8436.

* See Note 2, page 121.

† *Sometimes a less figure* requires to be taken, as in *the present example.*

* It is here contained 7 times in the dividend, but as it is found, on carrying the process further, that 7 is too many, 6, the next lower figure, is taken.

4. Multiply the complete divisor by the last figure of the root; subtract the product from the dividend; and to any remainder, annex the next period of three figures, to form a new dividend.

Here, the divisor, 8436, is multiplied by 6, the last figure of the root, and the product is subtracted from the dividend, 53453: to the remainder, 2837, is brought down the next period of three figures, forming the new dividend, 2837547.

5. Form a new DIVISOR as follows:—

Place the *square* of the last figure of the root under the previous divisor; add it to the three lines of figures above it; and to the amount annex two nothings, to form a partial divisor.

Here, 36, the square of 6, the last figure of the root, is placed below the complete divisor, 8436; it is then added to the three lines above it—namely, 8436, 36, and 900—and their amount, with two nothings annexed, forms the partial divisor, 940800.

Find how often the partial divisor is contained in the dividend; and, as already described in paragraph 3, place the quotient as the next figure of the required root: then multiply the *previous* figures of the root by this quotient, and the product by 30, placing the result below the partial divisor: also place the *square* of the quotient (that is, the square of the figure just annexed to the root) below the last product; then add the three sums together, and their amount is the *complete* divisor.

The partial divisor is contained 3 times in the dividend; the quotient, 3, is therefore placed as the next figure of the root: the root already found, 56, is then multiplied by the 3, and the product by 30, making 5040, which is placed under the partial divisor. The *square* of the quotient, 3—namely, 9—is placed below the last product, 5040. The three sums, 940800, 5040, and 9, are then added together, forming the complete divisor, 945849.

6. Multiply the complete divisor, and so on, as already described in paragraph 4.

7. Proceed to form new divisors according to paragraph 5; and go on with the rest of the process according to paragraph 4, till all the periods have been brought down and operated upon; when, if there is no remainder, the extraction of the root is completed.

Here, the divisor, 945849, is multiplied by 3, the last figure of the root, and the product is subtracted from the dividend, 2837547: there being no remainder, and no more periods to bring down, the extraction of the root is completed.

NOTE 1.—When there is a remainder after all the periods have been brought down, annex a period of three nothings, to form a new dividend; and then proceed with the further extraction of the root: the figure of the root thus obtained is a decimal. The process may be carried to any degree of minuteness by annexing more nothings.

There are always as many figures in the root as there are periods in the given number; and those figures are decimals in the root, which are extracted from the decimals in the given number.

NOTE 2.—If at any time, on bringing down a new period to form a dividend, the *partial* divisor is found to be greater than the dividend, a nothing must be placed in the root, and two nothings annexed to the partial divisor: the next period is then brought down, and annexed to the dividend, and the extraction of a new figure of the root proceeded with as before.

NOTE 3.—The cube root of a fraction is found by extracting the roots of the numerator and denominator.

Exercises.

Required the cube root of the following numbers :—

1. 110592, . . .	Ans. 48	4. 219365327791,	Ans. 6031
2. 373248, . .	" 72	5. 11, . . .	" 2·22398
3. 843908625, . .	" 945	6. 3·539, .	" 1·52391305

7. A box, whose length, breadth, and depth are equal, contains 216 cubic feet; what are its dimensions? . . Ans. 6 feet each way.

8. A person has a box 5 feet long, 4 feet broad, and 6¼ feet deep, and wishes another box to contain the same number of cubic feet, whose length, breadth, and depth shall be equal; what are the required dimensions? . . . . Ans. 5 feet each.

GLOBES BEAR THE SAME PROPORTION to each other as the cubes of their diameters: hence to find the diameter of a globe that shall contain, for instance, 8 times more than another whose diameter is 2 feet, multiply the cube of 2 feet, the given diameter, by 8, and extract the cube root of the product.

CUBES are in proportion to each other as the cubes of a side of each.

9. If a globe, whose diameter is 4 inches, weigh 5 lbs., what is the diameter of another globe which weighs 40 lbs.? . Ans. 8 inches.

10. If the side of a box, whose length, breadth, and depth are equal, is 4 feet long, what is the length of a side of another box of a cubical form, that contains 27 times as many cubic feet? . Ans. 12 feet.

EXERCISES ON THE SQUARE ROOT—*continued from page* 119.

THE AREA OF ONE CIRCLE bears the same proportion to the area of another, as the *squares* of their respective diameters.

21. The paving of a circular space, 50 feet in diameter, costs £74, 16*s*. 8*d*.; what will the paving of another, 120 feet in diameter, cost at the same rate? . . . . . . Ans. £431, 0*s*. 9½*d*. $\frac{2}{5}$

22. If a pipe, whose diameter is 1·5 inches, fill a cistern in 5 hours, in what time will another, whose diameter is 3·5 inches, fill the same? . . . . Ans. 55 min. 6 $\frac{6}{49}$ sec.

23. If a sluice, 3 feet in diameter, draw off the water in a pond in 20 hours, what must be the diameter of another that would draw it off in one-fifth of the time? . . . . . Ans. 6·7082 feet.

IN A RIGHT-ANGLED TRIANGLE, the square of the hypotenuse, or longest side, is equal to the amount of the squares of both the other two sides—that is, of the base and the perpendicular: hence, the length of the hypotenuse is ascertained by extracting the square root of the amount of the squares of the base and the perpendicular.

24. The base of a right-angled triangle is 342 feet, and the perpendicular 475 ft.; what is the length of the hypotenuse? Ans. 585·311028 ft.

25. The height of a wall is 31 ft. and the breadth of a ditch surrounding it, 24 ft.; what must be the length of a ladder that will reach from the edge of the ditch to the top of the wall? Ans. 39·20459 ft.

26. Close by a river, rises a precipice to the height of 261 feet, and a line, reaching from its top to the opposite bank of the river, measures 582 feet; what is the breadth of the river? Ans. 520·19515 ft.

SERIES OF NUMBERS.

A SERIES is a succession of numbers that increase or decrease according to some defined principle.

A series of numbers that succeed one another at a uniform rate of increase or decrease, called the common difference, is termed an Arithmetical Progression; as—4, 5, 6, 7, 8, which increase by 1 at each step; 3, 6, 9, 12, which increase by 3; and 12, 8, 4, which decrease by 4.

A series that advances or decreases by the multiplication of a given number at each step, is termed a Geometrical Progression; as—5, 15, 45, which increases by the multiplication of each number in succession by 3.

The great difference between the two kinds of progression, is exemplified in the following two lines; the number 3 being *added* in the one case, and being used as the *multiplier* in the other:—

Arithmetical Progression—5, 8, 11, 14, 17
Geometrical Progression—5, 15, 45, 135, 405

The first and last terms of any series are called the *extremes;* and the others, the *means.*

THE following popular story affords an instance of the rapid manner in which numbers increase by geometrical progression:—A gentleman who was extremely fond of horses, and did not grudge to give the highest prices for them, was one day called upon by a horse-dealer, who shewed him a horse that he thought superior to any he had seen before. He mounted him, and found his paces excellent; and, though full of spirit, he was gentle and tractable as could be wished. So many perfections delighted the gentleman, and he demanded the price. The horse-dealer answered, two hundred guineas; the gentleman, although he admired the horse, would not consent to give the sum, and they were on the point of parting, when the gentleman said: 'Is there no possible way of our agreeing, for I would give you anything in reason for such a horse?' 'Why,' replied the dealer, who was a shrewd fellow, and perfectly understood calculation, 'if you do not like to give me two hundred guineas, will you give me a farthing for the first nail the horse has in his shoe, two farthings for the second, four for the third, and so on; doubling the sum for every nail, of which there are twenty-four in his shoes?' The gentleman gladly accepted the condition, and ordered the horse to be led away to his stables. The horse-dealer now added: 'I do not mean to tie you down to this last proposal, which, upon consideration, you may like as little as the first: all that I require is, that if you are dissatisfied with your bargain, you will promise to pay me down the two hundred guineas which I first asked.' This the gentleman willingly agreed to, and then called the steward to calculate the sum. The steward sat down with his pen and ink, and, after some time, informed him that the sum amounted to seventeen thousand, four hundred and seventy-six pounds, five shillings, and fourpence! The gentleman was much surprised; but when, upon examination, he found it no more than the truth, he was very glad to compound for his foolish agreement, by giving the horse-dealer the two hundred guineas, and dismissing him. Thus, what would have amounted to no more than 12*s.* 6*d.* by arithmetical progression, was upwards of £17,000 by geometrical progression, in which only 2 was the multiplier.

DUODECIMAL MULTIPLICATION.

DUODECIMAL MULTIPLICATION is that which is employed in the measurement of walls, flooring, &c.; and solid bodies, such as logs of wood, in which feet, inches, and their subdivisions are multiplied together to ascertain the required dimensions.

It is so named from *duodecim*, a Latin word, signifying twelve, because in multiplying the feet, inches, &c., the number *carried* from one denomination to another is 12.

RULE.—1. Place the *feet* of the multiplier below the lowest denomination of the multiplicand; the inches, one place further to the right; the seconds, beyond the inches; and so on.

2. Multiply each denomination of the given quantity in succession, by the *feet* of the multiplier, as in Compound Multiplication; the *twelves* being taken out of each product, and carried to the denomination above it, and the remainder written below the denomination multiplied.

3. Multiply each denomination in the same way, by the *inches* of the multiplier, carrying the twelves as before, and writing the remainders of each product one place further to the right than those in the previous line; the first figure placed in the product being thus written immediately below the *inches* of the multiplier.

4. Multiply in the same way by the seconds, and so on; always writing the remainders in each new line of products, one place further to the right than those in the previous line—the first figure placed in each line, being thus written immediately below the figure used as the multiplier.

5. Then add all the products together for the answer, which is in *square* or *cubic* measure, as the case may be.

NOTE.—The product of feet, inches, &c. multiplied by feet, inches, &c. is expressed in *feet*, *firsts*, *seconds*, *thirds*; and so on. A *first* is 1-12th of a foot; a *second*, 1-12th of a first, &c. each denomination being 1-12th of that preceding it: each lower denomination is written a place further to the right than the one above it.

In multiplying by inches, seconds, &c. each *product* is of a lower denomination than the number multiplied, as shewn below, and must be converted to the highest denomination that it admits of, before being written down. Thus, in multiplying 346 feet by 4 seconds, the product is 1384 seconds (feet multiplied by seconds producing seconds); these are therefore divided by 12, to convert them to *firsts*; the answer is 115 firsts, and 4 seconds over; the 115 firsts are then divided by 12, to convert them to feet, and the answer is 9 feet and 7 firsts.

The following are the duodecimal divisions of feet, used to express the product of a multiplication:—

1 Foot . . . = 12 firsts.	1 Third, marked thus 1‴ = 12 fourths.
1 First, marked thus 1′ = 12 seconds.	1 Fourth 〃 〃 1⁗ = 12 fifths.
1 Second 〃 〃 1″ = 12 thirds.	1 Fifth 〃 〃 1′′′′′ = 12 sixths.

The result of multiplying by feet, inches, and seconds, is as follows:—

Feet	multiplied	by	feet	give	feet.
〃	〃	〃	inches	〃	firsts.
〃	〃	〃	seconds	〃	seconds.
Inches	〃	〃	feet	〃	firsts.
〃	〃	〃	inches	〃	seconds.
〃	〃	〃	seconds	〃	thirds.
Seconds	〃	〃	seconds	〃	fourths.

Examples.—Multiply 1 ft. 8 in. 4 sec. by 1 ft. 2 in. 6 sec.; and 346 ft. 6 in. 4 sec. by 2 ft. 3 in. 4 sec.

(1.)

	ft.	*in.*	*sec.*		
	1	8	4		
			1	2	6
	1	8	4		
		3	4	8	
			10	2	
sq.ft.	2	0′	6″	10‴	

(2.)

	ft.	*in.*	*sec.*		
	346	6	5		
			2	3	4
	693	0	10		
	86	7	7	3	
	9	7	6	1	8
sq.ft.	789	3′	11″	4‴	8⁗

Here it must be remembered, in multiplying by inches and seconds, that the products are of lower denominations than the numbers multiplied, and must be converted to their highest denominations before the figures are written in the product: thus, in example 2, the 346 feet multiplied by 3 inches produce firsts, which must be converted into feet; and the 346 feet multiplied by 4 seconds produce seconds, which must be converted into firsts, and then into feet, as explained in the note, page 124, before the figures are written in the product.

NOTE.—IN SQUARE MEASURE, the answer is in *square* feet, firsts, seconds, &c.: in expressing the product of a multiplication, the denomination next to feet is often erroneously termed *inches*; but although the number stands below the inches of the multiplicand, it does not express inches but *firsts*; thus, in example 2, the figure 3 of the product under inches, means 3 firsts, and is equal to 36 square inches, or 3-12th of a square foot, which contains 144 square inches: if the 3 had been square inches, it would have been in value only 1-48th of a square foot. IN CUBIC MEASURE, the answer is in *cubic* feet.

WHEN THE NUMBER OF FEET IS LARGE, it is often the most convenient method to multiply by the feet of the multiplier, and then to take aliquot parts for the inches, &c.—thus, for 6 inches, take the ½ of 1 foot; and so on.

THE AREA OR SQUARE CONTENTS of any surface such as a wall, or field, &c. are measured by multiplying its length by its breadth.

THE SOLID OR CUBIC CONTENTS of bodies, such as a log of wood, a block of marble, &c. are measured by multiplying their length by their breadth, and the product by the thickness or depth.

Exercises.—Find the area of a surface whose dimensions are—

	Length.		Breadth.		
1.	4 ft. 2 in.	by	3 ft. 7 in.	. . . Ans.	14 ft. 11′ 2″
2.	7 „ 8 „	„	5 „ 4 „	. . . „	40 „ 10′ 8″
3.	9 „ 5 „	„	3 „ 10 „	. . . . „	36 „ 1′ 2″
4.	11 „ 7 „	„	9 „ 8 „	. . . „	111 „ 11′ 8″
5.	13 „ 5 „ 4″	„	7 „ 4 „	. . . . „	98 „ 7′ 1″ 4‴
6.	7 „ 6 „ 9″	„	3 „ 7 „	. . . „	27 „ 1′ 2′ 3‴

7. How many square feet of flooring in a room 24 feet 7 inches long, and 16 feet 4 inches broad? . . . . . . Ans. 401 ft. 6′ 4″

8. Required the area of a plank, whose length is 20 feet 3 inches, and breadth 20 inches, ... Ans. 33 ft. 9′

9. How many square feet in a board 41 feet 8 inches long, 2 feet 8 in. broad at one end, and 1 foot 10 in. at the other?* Ans. 93 ft. 9′

* When the breadth is not the same throughout, the mean or average breadth is taken.

10. How many solid feet in a log of wood 20 feet long, 18 inches broad, and 14 inches thick? . . . . Ans. 35 ft.

11. How many solid feet of air in a room 68 feet 10 inches long, 35 feet 7 inches broad, and 20 ft. 3 in. high? Ans. 49598 ft. 8′ 7″ 6‴

12. How many solid feet in a stone 13 feet 8 inches long, 7 feet 9 inches broad, and 3 feet 11 inches thick? . . Ans. 414 ft. 10′ 1″

EXCHANGE, OR CONVERSION OF MONEY.

EXCHANGE is the conversion of sums in the money of one country, into equivalent sums in the money of another country.

Every nation has its own peculiar money: that of the United Kingdom, as already noticed, consisting of pounds, shillings, and pence, established at a certain standard value, known by the name *sterling*.

BRITISH COLONIES.—In Canada, Nova Scotia, and other British colonies in America, it is usual to reckon money also by pounds, shillings, and pence; but the value fluctuates, and to distinguish it from sterling, it is called *currency*. One of the most common standards is that of Halifax, according to which a pound sterling is considered equivalent to about 25 shillings currency, and a shilling sterling to 1*s.* 3*d.* currency.

TO CONVERT STERLING MONEY into currency, add a fourth to the sum in sterling; and to convert currency into sterling, deduct a fifth from the sum in currency.

UNITED STATES.—In the United States of North America, the standard money is *dollars* and *cents*. Each dollar contains 100 cents, and is equal to about 4*s.* 2*d.* sterling: the cent is equal to a half-penny sterling. The sign of the dollar is $. Accounts in dollars and cents are added as follows:—

	$	C.
	5	15
	7	00
	49	06
	11	83
Ans.	73	04

The cent column is added as in Simple Addition; the *tens* of the second row are carried to the dollars, which are then added in the usual way.

TO CONVERT BRITISH into United States Money, reduce the given sum to half-pence, which reckon as so many cents; then point off the two last figures as the cents of the answer, and the rest are dollars: thus, £1 is 480 half-pence or cents; and, on pointing off the two last figures, the sum is read as $4·80, or 4 dollars 80 cents.

TO CONVERT DOLLARS AND CENTS into British money, multiply 4/2 by the number of dollars, and for every cent reckon a half-penny; thus, 5 dollars 40 cents is equal to 4*s.* 2*d.* × 5 = £1, 0*s.* 10*d.*; and adding 40 half-pence, or 1*s.* 8*d.* for the cents, the answer is £1, 2*s.* 6*d.*

FRANCE.—In France, the standard money is *francs* and *centimes*, or cents. Each franc contains 100 centimes. The franc is equal to about 10*d.** in English money; therefore, 10 centimes are equal to an English penny. Accounts in French money are added as follows:—

* 9½*d.* more exactly.

	F.	C.
	15	25
	2	50
	9	60
	25	00
Ans.	52	35

The cent column is added as in Simple Addition; the tens of the second row are carried to the francs, which are then added in the usual way.

TO CONVERT BRITISH into French money, reduce the given sum to pence, and point off the last figure of the product: the answer is francs, and the figure pointed off, is so many 10ths of a franc.

TO CONVERT FRENCH into British money, reckon the francs as so many shillings, and then deduct a sixth from the amount: the cents may be reckoned at the corresponding proportion of 1*s.*

Exercises.

1. If I pay £12, 10*s.* for ten head of oxen, how many dollars does each cost me? . . . . Ans. 6 dollars each.

2. A tradesman in Paris earns 2 francs 75 centimes per day; how much is that for a year of 313 working-days? . Ans. £35, 17*s.* 3½*d.*

3. A gentleman buys in Paris, books and stationery to the amount of 167 francs; how much is that in British money? Ans. £6, 19*s.* 2*d.*

4. I paid 18 francs 50 cents for a pair of boots in Calais, which were said to be worth 25*s.* in London; how much did I save by buying the boots in France instead of in England? . . . Ans. 9*s.* 7*d.*

5. An emigrant to Canada paid for 100 acres of land, £36 currency; for a house, £7 currency; and for clearing 2 acres, £3, 12*s.* currency; what is the amount in sterling money? . . Ans. £37, 5*s.* 7*d.* $\frac{4}{5}$

6. If the expense of travelling from New York to Cincinnati be $35, 50 cents, what is the expense in sterling money? Ans. £7, 7*s.* 11*d.*

7. A farmer in Illinois sows 250 acres of land, at an expense of $1, 75 cents per acre; what does the whole cost in sterling money? Ans. £91, 2*s.* 11*d.*

8. If I purchase 1200 acres of land, at the rate of 16*s.* currency per acre, how much is the whole in sterling money? . . Ans. £768

NOTE.—With respect to the above, and all other foreign monies, there is usually a *premium* for or against, in making the exchange, which requires to be taken into account in actual business. Thus, a person taking a sovereign to Paris, may in reality get 25 instead of 24 francs for it, or a premium of 1 franc. In purchasing bills in the colonies drawn on parties in England, a premium is generally exacted according to the demand for such bills; therefore, although £125 currency is equal to £100 sterling, it may happen that the purchaser of a £100 bill on England may have to pay for it £130 or £135. When no premium is exacted, the course of exchange is said to be at *par.*

BOOK-KEEPING AND ACCOUNTS.*

BOOK-KEEPING is the art of recording and classifying a merchant's or tradesman's daily transactions, and of keeping an account of his property and debts.

A merchant's books ought to exhibit clearly the whole amount of his property, with the particulars of which it is composed; and also the amount of his debts.

The following are the most important of the books used in Book-keeping:—

DAY-BOOK,	for Goods *sold* on credit.
INVOICE-BOOK,	" Goods *bought* on credit.
CASH-BOOK,	" Cash received and paid. Discount received and allowed.
BILL-BOOK,	" Bills receivable and payable.
LEDGER,	to contain an abstract of the other books.

Besides these, a Stock-book, to contain a list of stock on hand: an Account-book, to contain a list of accounts; and various others, are employed in business.

* This subject is treated of at length in a separate work on Book-keeping in the Educational Course.

DAY-BOOK.

The purpose of the Day-Book is to keep a daily account of all goods *sold on credit.* The names and addresses of the persons to whom they are sold, with a description of the goods and their prices, and any other charges, are entered in the Day-Book, as shewn in the following examples:—

January 1, 1854.

		£	s.	d.	£	s.	d.
	George Innes, Liverpool. *Dr.*						
3	To 20 Yards Black Silk .... @ 3/6	3	10	0			
	,, 12 ,, Velvet .... ,, 11/	6	12	0			
	,, 15 ,, Linen .... ,, 2/6	1	17	6			
	Wrapper and packing	0	3	6			
	Commission	0	12	0	12	15	0
	"Dr. To" is a contraction for "Debtor To."						
	—*6th.*—						
	James Brown, Edinburgh.						
6	To 10 lbs. Sugar .... @ 6½d.	0	5	5			
	,, 2 ,, Tea .... ,, 4/6	0	9	0	0	14	5
	The numbers 3 and 6 on the margin indicate the pages of the Ledger to which the entries are posted or copied. See Ledger.						

CASH-BOOK.

In the Cash-Book is kept an account of all cash received and paid, and of discount received and allowed. Two pages are always required for the entries: the left-hand page for entering the cash you receive, and the discount

Cash Received.

		Discount			Cash.		
	Jan. 1, 1854.						
	Cash on hand				20	5	6
2	Goods—Cash Sales				7	10	6
	This is an entry for goods sold for ready money.						
4	James Arnold, London	18	10	0	351	0	0
	Here J. Arnold settles his account by paying £351; the sum of £18, 10s. being allowed to him as discount.						
					378	16	0
	—*Jan.* 2.—						
8	Union Bank. (This sum is received by you from the bank.)...				150	0	0
7	James Edwards, Princes Street	0	3	3	3	5	6
10	Bills Receivable, No. 1, discounted, £50	0	4	0	49	16	0
	This bill is discounted by you at the bank.						
					581	17	6
	—*Jan.* 31.—						
6	James Pringle, Dublin	24	15	6	470	9	0
2	Goods—Cash Sales				6	15	0
	BALANCING THE CASH-BOOK.—To ascertain that the sums have been entered correctly, the Cash-Book requires to be balanced daily, as on the other side.				1059	1	6

BILL-BOOK.—In this book is kept, in one portion, an account of all "Bills Receivable"—that is, bills of which you have to receive payment—and in

INVOICE-BOOK.

This book is used for keeping an account of all goods *bought on credit*. It is so called because the entries made in it are copied from the *invoices* usually sent along with the goods. The names and addresses of the persons from whom they are bought, with a description of the goods and their prices, and any other charges, are entered in the Invoice-Book, as below:—thus,

January 1, 1854.

			£	s.	d.	£	s.	d.
	James Stewart & Co., Leeds.	*Cr.*						
	By 9 Pieces Black Cloth, 180 yards*...............	@ 13/7	122	5	0			
7	,, 3 ,, Brown do. 80 ,, ...	,, 14/5	57	13	4			
	,, 4 ,, Olive do. 80 ,, ...	,, 14/10	59	6	8			
	,, 5 ,, Black do. 100 ,, ...	,, 15/2	75	16	8			
	Wrapper and packing...		0	9	6	315	11	2
	* that is, 9 pieces measuring 180 yards, at 13s. 7d. a yard. "Cr. By" is a contraction for "Creditor By."							
	—*20th.*—							
12	John Miller, Edinburgh.							
	By Goods, as per Invoice, January 19.............................					33	15	6
	In practice, instead of copying the particulars of the invoices, it is often more convenient to enter the amount only, with the date of the invoice.							

CASH-BOOK.

allowed *by* you; the right-hand page for the cash you pay, and the discount allowed *to* you. Each page is ruled with double money columns; the inner columns being used for the discount, the outer for the cash.

Cash Paid.

		Discount			Cash.		
	Jan. 1, 1854.						
8	Union Bank. (This sum is paid by you into the bank.)				250	0	0
2	Goods—Cash Purchases..				6	15	0
	This is an entry for goods bought for ready money.						
12	Trade Expenses—Carriages				0	17	9
4	James Watson, Edinburgh	5	0	0	95	0	0
	You settle J. Watson's a/c by paying him £95, discount £5 being allowed to you.						
	Cash on hand, £26, 3s. 3d.*				"	"	"
	* This is marked here to shew that the two sides balance when the cash on hand is taken into account.				352	12	9
	—*Jan* 2.—						
11	Bills Payable, No. 1..				200	0	0
	Cash on hand, £29, 4s. 9d.				"	"	"
					552	12	9
	—*Jan.* 31.—						
8	Union Bank..				500	0	0
	Cash on hand..				6	8	9
	The cash on hand is noted every day as above; but the sum is not extended to the money columns till January 31, when the Cash-Book is finally balanced for the month.				1059	1	6

another portion, "Bills Payable"—that is, bills which you have to pay when they become due.

THE LEDGER.

In the Ledger is contained an abstract of all the entries made in the other books.

The entries dispersed throughout the Day-Book, Invoice-Book, Cash-Book, and Bill-Book, are collected together in the Ledger, and arranged in the order of their dates, under the names of the various persons to whom they belong.

A page, or such portion of a page as is likely to be required, is assigned to every person's account; and each page being ruled with *Dr.* and *Cr.* columns, the amounts of all the *Dr.* entries belonging to each person, are copied one by one into the *Dr.* sides, and the amounts of all the *Cr.* entries into the *Cr.* sides, of the respective accounts in the Ledger. An index is made of all the names entered.

The copying of these entries into the Ledger is termed *posting*. The pages in the Ledger to which they are posted are marked on the margin opposite the various entries in the different books.

Examples.

James Brown, 75 George Street.

Dr. *Cr.*

				£	s.	d.					£	s.	d.
1854.							1854.						
Jan.	6	To Goods..............	1	0	14	5	Jan.	10	By Cash...............	1	0	14	0
								,,	,, Discount.........	,,	0	0	5
				0	14	5					0	14	5

George Innes, Liverpool.

Dr. *Cr.*

				£	s.	d.					£	s.	d.
1854.							1854.						
Jan.	1	To Goods...............	1	12	15	0	Jan.	9	By Cash................	1	12	15	0
Feb.	6	,, Do.	5	67	3	9	Feb.	25	,, Do.	3	65	10	0
Mar.	4	,, Do.	11	175	11	0			,, Discount..........	,,	1	13	9
	22	,, Cash................	6	20	0	0	Mar.	11	,, Bill due April 14	1	175	11	0
	31	,, Bill due May 2..	3	80	0	0		20	,, Goods..............	6	100	0	0
				355	9	9					355	9	9

The entries of goods on the *Dr.* side are posted from the Day-Book, and on the *Cr.* side from the Invoice-Book. The entries of cash and bills are posted from the Cash and Bill-Books.

The following is a form of the Ledger often used by retail dealers, in which the *Dr.* and *Cr.* money columns are placed together, instead of on the opposite sides of each page, as in the Ledger above. The object of this is to give more space for writing the particulars of the entries:—

John Simpson, Esq. George Street. *Dr.* *Cr.*

				£	s.	d.	£	s.	d.
1854.									
Jan.	8	To 1 cwt. Sago..	2	0	18	0			
	,,	,, 1 Box Preserved Fruit..	,,	0	12	6			
Feb.	11	,, 12 lbs. Wax Candles.................................. 2/6	7	1	10	0			
Mar.	7	,, 6 lbs. Congou.. 4/6	9	1	7	0			
April	6	By Cash..	10				2	0	0
May	1	To Half cwt. Sago...................................... 18/	12	0	9	0			
June	16	,, 1 Cheese..	13	3	11	0			
	,,	By Cash..	,,				6	7	6
				8	7	6	8	7	6

Balance Sheet.—To ascertain the state of affairs at the end of the year, or at any other convenient time, it is necessary to draw out a "Balance Sheet"—that is, a statement shewing how much is owing to you, and by you; also the amount of cash, bills, and stock on hand, and what is the balance, if any, in your favour.

Dr. **Balance Sheet of John Adams.** *Cr.*

1854.				£	s.	d.	1854.				£	s.	d.
Dec.	31	To Accounts due by J. Adams..£200 less discount 10		190	0	0	Dec.	31	By Goods on hand..		410	0	0
									,, Shop Furniture.	12	95	0	0
		,, Bills due by J. A.		160	0	0			,, Accounts due to J. Adams £300 less discount 15		285	0	0
		,, Balance, nett capital		1150	0	0			,, Bills due to J. A.		200	0	0
		The sums-total only of the accounts are entered in the Balance Sheet: the particulars are given in the "Account-Book."							,, Cash in Bank...	12	500	0	0
									,, Do. on hand, per Cash-Book.....		10	0	0
				1500	0	0					1500	0	0

Profit.—To ascertain the profit, if any, that has been gained during the year, open an account under the head of "Profit," or "Profit and Loss," and enter as follows:—

Dr. **PROFIT (or, Profit and Loss).** *Cr.*

1854.				£	s.	d.	1854.				£	s.	d.
Jan.	1	To Capital at this date..................	14	1000	0	0	Dec.	31	By Capital at this date..............	15	1150	0	0
Dec.	31	,, Profit............... (Including £50 of interest at 5 per cent. on the capital).	,,	400	0	0			,, Cash to J. A. in 1854..........	14	250	0	0
				1400	0	0					1400	0	0

The *nett* profit is shewn above, after paying trade expenses, &c. By adding to the nett profit the amount of expenses and bad debts, the *gross* profit will be ascertained; thus:—

Nett Profit..	£350	0	0
Trade Expenses, £100. Bad Debts, £50, as per Ledger.....	150	0	0
Gross profits..............................	500	0	0

ACCOUNTS.

An Account is a statement shewing the amount due by one person to another for goods, cash, &c. Accounts are kept under their several titles in the Ledger, from which they are copied when required.

Account copied from a *Wholesale* Ledger.

Mr George Knight, *Birmingham*. To **Hamilton & Boyd**, *London*.

1854.					
Mar.	4	Goods..	278	7	0
	11	do.	19	17	7
	26	do.	664	11	8
			962	16	3

In rendering this account, it is unnecessary to give the particulars of the entries. When the goods were forwarded, *invoices*, containing the particulars, were either sent along with the goods, or separately by post; and in making out the account, only the dates and sums require to be stated.

Account copied from a *Retail* Ledger.

Mr JOHN ALISON, *Frederick Street.*

To JOHN ADAMS, *Edinburgh.*

				£	s.	d.
1854.						
Feb.	20	2 Pair Blankets	17/6	1	15	0
	,,	10 Yards Superfine Black Cloth	18/6	9	5	0
	,,	5 ,, Doeskin	5/6	1	7	6
Mar.	12	6 lbs. Green Tea	5/6	1	13	0
	,,	8 ,, Black do.	4/	1	12	0
	,,	6 ,, Loaf Sugar	8d.	0	4	0
				15	16	6

In copying this account, all the particulars require to be given, as no invoice or account of these was sent when the goods were got. The Ledger contains only the sums-total of each entry; the particulars are ascertained by referring to the Day-Book.

If a note of the account is again sent to John Alison that payment may be got, it is written out in this way:—

Mr JOHN ALISON, *Frederick Street.*

To JOHN ADAMS, *Edinburgh.*

1854.					
Mar.	12	Account rendered	15	16	6
		July 1 Paid JOHN ADAMS.			

INVOICES.

AN INVOICE* is a list or account of the particulars and prices of goods, &c. that have been sold on a certain day by one person to another. The following is an example:—

MANCHESTER, *January* 1, 1854.

Mr JOHN ADAMS,
Edinburgh.

Bought of EDWARD JOHNSTON & Co.

26 Pieces Printed Cotton, 825 yards ... 7d.	24	1	3
2 ,, do. do. 120 ,, ... 8d.	4	0	0
12 ,, Twilled do. 504 ,, ... 7d.	14	14	0
Wrapper	0	2	6
per Rail.	42	17	9

INVOICES to be written out and priced.

1. *London*, January 23, 1854.—Robert Dixon, Esq. To James Turner, carpenter, 2 days' work of a man, at 4*s.* 6*d.*; wood, 7*s.* 6*d.*; nails, 1*s.* 3½*d.* . . . . Ans. 17*s.* 9½*d.*

2. *Edinburgh*, February 27.—Mr George Thomson bought of Charles Draper and Co. 37 yards calico, at 7¼*d.*; 46 yards shirting, at 9½*d.*; 35 yards chintz, at 1*s.* 3*d.*; 2 pieces print, 57 yards, at 10½*d.*; 1 piece linen, 28¾ yards, at 2*s.* 3½*d.*; 29 yards Welsh flannel, at 1*s.* 8*d.*
Ans. £13, 6*s.* 7¼*d.* ½

* Sometimes called a *Bill of Parcels.*

INVOICES—*continued.*

3. *London*, January 17, 1854.—Mr T. Evans, To J. Rankin, shoemaker, 1 pair shoes, 10*s*. 6*d*.; 1 pair Wellington boots, 27*s*.; 1 pair straps, 6*d*. . . . . Ans. £1, 18*s*.

4. *Canterbury*, February 13, 1854.—Mr James Fife bought of John Arnot, 1½ lb. black tea, at 4*s*. 4*d*.; 14¼ lbs. refined sugar, at 7½*d*.; 16 lbs. raw sugar, at 5½*d*.; 6 lbs. of coffee, at 1*s*. 9*d*.; 8¼ lbs. soap, at 7¼*d*.; 7½ lbs. rice, at 3½*d*. . . . . . . Ans. £2, 0*s*. 4¾*d*.

5. *Manchester*, March 1, 1854.—Messrs W. Allan and Co. bought of John Hutchison and Co. 15 pieces book-muslin, at 9*s*. 6*d*.; 26 pieces jaconet, at 13*s*. 6*d*.; 27 pieces print, at 23*s*. 6*d*.; 36 pieces print, at 18*s*. 5*d*.; 6 pieces bombazine, 360 yards, at 1*s*. 4½*d*. . Ans. £114, 6*s*.

6. *Liverpool*, May 5, 1854.—Messrs George Thomson and Co. bought of Peter Morrison, 12 damask table-cloths, 6-4ths, at 4*s*. 6*d*.; 12 do. 8-4ths, at 7*s*. 6*d*.; 6 do. 10-12ths, at 12*s*. 6*d*.; 3 pieces 5-8th diaper, 134 yds. at 10½*d*.; 3 pieces do. 7-8ths, 127 yds. at 1*s*. 4*d*. . Ans. £25, 5*s*. 7*d*.

7. *Edinburgh*, July 8, 1854.—Mr James Wilson, Greenock, To George Innes. 4¾ yards flannel, at 2*s*.; 3½ yards cotton, at 1*s*. 4½*d*.; 5 yards drab cassimere, at 8*s*. 6*d*.; 3 yards superfine black cloth, at 19*s*. 6*d*.; 1 umbrella, at 7*s*. 6*d*. . . . . Ans. £6, 2*s*. 9¾*d*.

8. *Glasgow*, August 10, 1854.—Messrs James Bruce and Co. To James Hutton. 15 pieces muslin, 150 yards, at 2*s*. 4*d*.; 4 pieces cambric, 60 yards, at 3*s*. 2*d*.; 24 pieces cotton, 450 yards, at 1*s*. 5*d*.; 75 yards figured poplin, at 2*s*. 8*d*. . . . . Ans £68, 17*s*. 6*d*.

9. *Leeds*, December 5, 1854.—Mr John Taylor, To John Dawson and Co. 76 yards drab cassimere, at 7*s*. 4*d*.; 45 yards superfine blue cloth, at 21*s*. 6*d*.; 84 yards superfine black cloth, at 23*s*. 4*d*.; 10¾ yards black cassimere, at 7*s*. 9*d*. . . . . . . Ans. £178, 8*s*. 1¾*d*.

10. *Glasgow*.—Messrs William Hamilton and Co. To Charles M'Vicar and Co. January 13, 1854, 12 pieces calico, 336 yds. at 8¼*d*.; 12 pieces sheeting, 685 yds. at 9½*d*. March 10, 18 pieces lawn muslin, at 12*s*. 6*d*.; 22 pieces lawn muslin, at 14*s*. 6*d*. April 20, 10 pieces checked muslin, at 14*s*. 3*d*.; 32 pieces print, at 21*s*. 6*d*. . . . Ans. £107, 7*s*. 9½*d*.

11. *Carlisle*, April 16, 1854.—Mr Thomas Jones bought of Alexander Dunlop and Co. 6 pieces common Scotch carpeting, 437 yards, at 2*s*. 3*d*.; 6 pieces superfine Scotch carpeting, 453 yds. at 3*s*. 3*d*.; 3 pieces Brussels carpeting, 221 yds. at 3*s*. 9*d*.; 4 pieces lobby carpeting, 315 yds. at 1*s*. 5*d*. and 4 pieces, 289 yds. at 1*s*. 8*d*. Ans. £210, 12*s*. 2*d*.

12. *Birmingham*, March 10, 1854.—Mr A. Bell bought of James Wright and Co. 6 doz. table-knives and forks, at 15*s*. 6*d*.; 6 doz. desserts to match, at 9*s*. 3*d*.; 6 pair carving-knives and forks to match, at 4*s*. 9*d*.; 2 doz. pen-knives, 16*s*. 9*d*.; 2 doz. pen-knives, double blades, 13*s*. 4*d*.; 3 doz. razors, 27*s*. 6*d*.; 4 doz. pair scissors, at 1*s*. 5*d*. a pair, ... Ans. £19, 7*s*. 8*d*.

13. *London*.—Mr John Price to A. Hall and Co. January 7, 1854, 2⅔ yds. superfine black cloth, at 18*s*. 9*d*.; 3¼ yds. superfine blue cloth, at 19*s*. 6*d*. February 16, 5¾ yds. silk velvet, at 13*s*. 4*d*.; 3½ yds. cotton velvet, at 2*s*. 6*d*. April 3, 3¾ yds. gros de Naples, at 2*s*. 9*d*.; 4⅝ yds. satin, at 7*s*. 9*d*. May 14, 3⅛ yds. superfine olive cloth, at 12*s*. 6*d*.; 5¾ yds. brown cloth, at 10*s*. 9*d*. . . . . Ans. £17, 9*s*. 8¾*d*.

BILLS.

A Bill is an agreement written on stamped paper, in which a debtor agrees to pay to his creditor on a certain day, a specified sum of money which he is owing to him.

Bills are used for the settlement of accounts or debts. They are drawn at various dates; but in trade the usual term is from two to nine months. A Bill is termed an 'Acceptance,' or a 'Promissory-note,' according to the form in which it is drawn out.

AN ACCEPTANCE.

£100. due Nov. 8. *London, 5th August* 1854.

Three months after date, pay to me or my order the sum of one hundred pounds, value received.

John Wilson.

To Mr Thomas Arnold, Merchant,
Strand, London.

The bill being drawn by Mr Wilson in this form, Mr Arnold, on whom it is drawn, *accepts* it, by writing his name either below that of Mr Wilson, or across the face of the writing; hence he is termed the acceptor of the bill. The banking-house at which the bill is payable is also sometimes stated by the acceptor.

If Mr Wilson, who is called the drawer of the bill, wishes to make use of it, he indorses the bill—that is, writes his name *across* the back of it; and thus it becomes negotiable paper. It may be paid away to a third party; and he indorsing it below Wilson's name, may pay it away to a fourth; and so on. Thus the bill may pass from hand to hand, on each occasion liquidating a debt of £100, till the day of payment by the original acceptor arrives, when it is duly presented by the last holder.

Discounting Bills.—Instead of running this course, the bill may at any period be *discounted* by a bill-broker or banker. The discounting of a bill consists in giving the money for it, less a certain sum for *interest*. When a bill for £100 for three months (or fourth part of a year) is discounted at 5 per cent. a charge equal to the fourth part of a year's interest is made by the discounter, and this is his profit for the loan of the money for that period.

Presentation for Acceptance.—'An Acceptance,' to render it complete, requires to be duly presented to the party on whom it is drawn, that he may *accept* of it. This is not necessary in a 'Promissory-note.'

Presentation for Payment.—All bills require to be presented for payment on the exact day they become due—that is, on the last *day of grace* (see next page): if not presented, they cease to have the peculiar privileges of bills, and become mere evidences of debt.

Noting and Protesting.—When a bill is not duly paid on presentation, the holder applies to a notary-public, who again presents the bill. If not paid, he *notes* its non-payment, and afterwards draws out a formal *protest* on stamped paper, that legal steps may be taken for recovering the amount. The bill should be *noted* on the day it falls due: the *protest* may be written out afterwards.

A PROMISSORY-NOTE.

£100. due Nov. 8. *London, 5th August* 1854.

Three months after date, I promise to pay to Mr James Brown, or order, the sum of one hundred pounds, value received.

Robert Hamilton.

No signature is written across the front of the promissory-note; it is complete in itself, and only requires to be indorsed by the holder of it (in the above *case*, James Brown) when he wishes to make use of it, or to pay it away. *Promissory-notes* are in every respect liable to the same regulations as *acceptances*.

Bills are sometimes drawn at *sight*, or at so many days after sight; thus—

£50. *London, 5th August* 1854.

Ten days after sight, pay to me or my order the sum of fifty pounds, value received.

John Thomson.

To Mr Thomas Jones, Liverpool.

A bill of this kind requires to be sent to the debtor to be *sighted;* which consists in the debtor—as, for instance, the above Mr Jones—accepting it by signing his name, and marking the day on which he has done so. The bill on being *indorsed* becomes a negotiable instrument, and on the third day after the day specified, it is presentable for payment.

Foreign bills of exchange are usually drawn at so many days after sight. They are of precisely the same nature as inland or home bills of exchange; but for the sake of security in transmission, they are drawn in sets of three.

FOREIGN BILL.

Exchange for £100 *sterling.* *Philadelphia, Jan.* 1, 1854.

No. 479. *Sixty days after sight of this* FIRST *of* EXCHANGE *(second and third of same tenor and date unpaid), pay to the order of John Robertson the sum of one hundred pounds sterling, value received.*

James Anderson.

To Messrs Brown and Jones,
Merchants, Liverpool.

This bill being indorsed by Mr Robertson, is transmitted to England, and is presented to *Messrs Brown and Jones to be sighted* by them: when it becomes due, it is *presented* to them for payment accordingly.

Days of Grace.—According to a practice of old-standing, bills are not presentable for payment till the third day after that which is specified for them to fall due. The three days allowed are called the *days of grace.* Thus a bill drawn on the 5th of August, at three months, is not legally due till noon of the 8th of November.

MISCELLANEOUS EXERCISES IN ARITHMETIC.

1. What does the rent of a house amount to from May 15 to January 20, at the rate of £15, 10*s.* a year? . . Ans. £10, 12*s.* $3\frac{3}{4}$*d.* $\frac{57}{73}$

2. Two travellers depart at the same time from the same place; the one goes 20 miles a day, the other, $23\frac{1}{2}$: how far will they be from one another at the end of 14 days, if they both travel in the same direction?—and how far, if in contrary directions? Ans. 49 miles; 609 miles.

3. A merchant failing in trade, owes A £500, 10*s.*; B, £350, 15*s.* 6*d.*; C, £245, 0*s.* $6\frac{1}{2}$*d.*; D, £56, 18*s.* $1\frac{1}{2}$*d.*; and E, £337, 1*s.* 8*d.*: he has in cash £192, 7*s.* 1*d.* and his effects amount to £739, 1*s.* $6\frac{3}{4}$*d.*: what will each creditor receive per pound, supposing his cash and effects delivered to them? . . . . . . Ans. 12. 6*d.* per pound.

4. Shipped on an adventure to Lisbon, 300 barrels of salmon, at £3, 18*s.* 6*d.*; 450 yards of linen, at 2*s.* $7\frac{1}{2}$*d.*; 1200 yards of broad cloth, at 16*s.* 4*d.*; paid for insurance and other charges, £53, 18*s.* 6*d.*; the nett proceeds, as per account sales, was £2463, 17*s.* 9*d.* What was the whole gain or loss?—gain or loss per cent.?

Ans. £193, 8*s.* gain on the whole; £8, 10*s.* $4\frac{1}{4}$*d.* $\frac{49897}{181639}$ gain per cent.

5. If 17 guineas be lost by the sale of 460 lbs. of raw silk, at £1, 6*s.* 4*d.* per lb. what was the prime cost per lb.?—and loss per cent.?

Ans. £1, 7*s.* $1\frac{1}{4}$*d.* $\frac{22}{115}$ per lb.; £2, 17*s.* $3\frac{0}{4}$*d.* $\frac{10572}{37411}$ loss per cent.

6. Bought paper at 17*s.* 6*d.* per ream; at what must I sell it per ream to gain £26 on 310 reams, and give six months' credit?

Ans. 19*s.* $7\frac{3}{4}$*d.* $\frac{82}{155}$

7. How many deals, 14 feet 6 inches long, and 9½ inches broad, will floor a room 70 feet by 30? . . . . Ans. 182$\frac{518}{551}$

8. In 1841, the population of Great Britain amounted to 18,813,786; in 1851, it was 21,121,967: what was the increase per cent. in the ten years? . . . . Ans. 12·27 nearly.

9. Three persons, A, B, and C, purchased a ship, of which A paid for $\frac{2}{9}$, B for $\frac{3}{5}$, and C paid £400; what part of the ship had C, and what did A and B pay? Ans. C had $\frac{8}{45}$; A paid £500; and B, £1350.

10. There are three boxes—the contents of one are 10,000 solid inches; of another, 16,656; and of the third, 20,000: required the side of a cubical box that will contain as much as all three, . Ans. 36 inches.

11. What will the digging of the foundation of a house 68 feet long, 33 broad, and 5 deep, come to, at 1*s.* 3*d.* per solid yard?
Ans. 415$\frac{5}{9}$ yards; £25, 19*s.* 5¼*d.* ⅓

12. A person observed the flash of a cannon 7 seconds before he heard the report; how far was the cannon distant, supposing that sound moves at the rate of 1142 feet per second? Ans. 1 m. 904 yds. 2 ft.

13. The garrison of Gibraltar, numbering 6000 men, has provisions for 16 weeks; it receives a re-inforcement of 400 men; how long will the provisions last? . . . . Ans. 15 weeks.

14. A has a quantity of pepper, weighing 7800 lbs. at 1*s.* 3½*d.* per lb. which he barters with B for equal quantities of ginger, at 7½*d.* per lb. and cinnamon, at 4*s.* 9*d.* per lb.; how many pounds of each did A receive? . . . . Ans. 1874 $\frac{18}{43}$ lbs. of each.

15. A and B engaged in trade with a capital of £4000, of which A's share was to B's as 7 : 5; at the end of 15 months their whole stock was increased to £5680: what was each partner's share of the gain? . . . . Ans. A, £980; B, £700

16. In 1851, the population of England and Wales was 17,927,609, and the total number of children at day-schools was 2,144,378; what was the ratio of scholars to the population?—and what percentage of the population was at school? Ans. Ratio, 1 in 8·36; Percentage, 11·96

17. In the same year, the population of Scotland was estimated at 2,888,742, and the number of children at schools was 368,517; what was the ratio of scholars to the population?—and what percentage of the population was at school? Ans. Ratio, 1 in 7·83; Percentage, 12·75

18. A tank can be filled with water by two separate pipes: by the first, it would be filled in 10 hours; by the second, in 8 hours. It is begun to be filled by the first alone; but when half full, the supply from the second pipe is turned on. What was the whole time of filling the tank? . . . . Ans. 7 hours 13⅓ min.

19. An architect employs 24 masons at 2*s.* 6*d.* 10 labourers at 1*s.* 8*d.* and 6 miners at 2*s.* for every working-day from Feb. 4 to Nov. 11; but reduces their wages during winter as follows:—masons, 2*s.*; labourers, 1*s.* 4*d.*; and miners, 1*s.* 8*d.* per day. Required the yearly income of each, ... Ans. A mason's income, £37, 6*s.*;
a labourer's, £24, 17*s.* 4*d.*; a miner's, £30, 1*s.* 8*d.*

20. If the pressure of air on the body of a man at the level of the sea, is 15 lbs. to the square inch; what is the total pressure he sustains, the surface of his body containing as many square feet as a board 135 inches long and 16 inches broad?
Ans. Total pressure, 14 tons, 9 cwts. 32 lbs.

APPENDIX.

DECIMAL MONEY.

A NEW SYSTEM OF RECKONING MONEY DECIMALLY, by subdividing the pound into tenths, hundredths, and thousandths, is at present under consideration by Government, with a view to its introduction, if found to be practicable.

The precise details of the proposed system are not yet determined upon, nor the names to be given to the new denominations of money, but the leading principle in the contemplated change is to divide the pound into 1000 parts, instead of, as at present, into 960 farthings, and to advance from one denomination to another, by *tens* (or combination of tens), as in simple numbers, instead of from 4 farthings to 1 penny, 12 pence to 1 shilling, 20 shillings to 1 pound. By this means, calculations in money will become as easy as those in simple numbers, and will be wrought by the rules of *Simple* Addition, Subtraction, &c.

It has been proposed by some, to divide the pound into 1000 parts, termed *mils*—10 mils to make one cent; 10 cents, 1 florin; and 10 florins, 1 pound. By others, it is proposed to term the 1000 parts into which the pound is to be divided, *cents*—100 cents to make 1 florin, and 10 florins, 1 pound.

As this latter plan is simpler than the other, we shall assume it to be the one adopted; and as the principle of dividing the pound into 1000 parts is the same in both, the mode of working questions will also be the same, whichever plan should ultimately be preferred.

THE FOLLOWING TABLE will shew the nature of the proposed decimal division of the pound. Both of the proposed plans are given.

	Florins.		Cents.
	1	=	100
£1 =	10	=	1000

or,

	Florins.		Cents.		Mils.
			1	=	10
	1	=	10	=	100
£1 =	10	=	100	=	1000

Accounts would thus be kept in pounds, florins, cents; and shillings, pence, and farthings would be disused.

As a cent is the 1000th part of a pound, whilst a farthing is the 960th part, it follows that a farthing is equal to $1\frac{1}{24}$ cent; and a cent to $\frac{24}{25}$ of a farthing—that is, $\frac{1}{25}$ less than a farthing; 25 cents, therefore, will be the same as 24 farthings. A florin, or 100 cents, is equal in value to 2*s*.

Besides florins and cents used in keeping accounts, various other coins, each representing so many cents, will be required. The coins under the new system may probably be nearly as follows:—

			s.	*d.*		Cents.
Gold	Sovereign or pound,				=	1000
"	½ do.				=	500
Silver	Florin,	=	2	0	=	100
"	½ do. or shilling,	=	1	0	=	50
"	¼ do. or present sixpence,	=	0	6	=	25
"	10 Cent-piece,	=	0	$2\frac{2}{5}$	=	10
Copper	5 Cent-piece,	=	0	$1\frac{1}{5}$	=	5
"	2 do.				=	2
"	1 do.	= $\frac{24}{25}$ of a farthing,			=	1

THE DECIMALS OF A POUND—that is, florins and cents—may be written in two ways; thus—

£	Florins.	Cents.		£ Cents.
36	9	75	or,	36 · 975

The first method may probably be used in merchants' books: the second, in which the 9 florins 75 cents are written as 975 cents, will be the most convenient in calculations; all that is necessary to distinguish the cents from the pounds, being the placing a decimal point between them.

In adding, subtracting, multiplying, dividing, &c. sums of money, the process will be exactly the same as in simple numbers, except that it will be necessary, in reading the figures, to attend to the placing of the decimal point in all cases before the *third* figure from the right when the given number is cents, and before the *first* figure on the right when the number is florins, to distinguish the florins and cents from the pounds.

It is to be observed that *three* figures are used to express the decimals of a pound—thus, ·346: the first decimal means florins, and the two last cents; or all the three figures may be read as cents.

In any number written as cents, the three last figures are read as cents (or florins and cents), and all the rest as pounds, the decimal point being placed before the third figure from the right: thus, 43624 cents is read as £43, 6 florins, 24 cents, or as £43, 624 cents.

Again, in any number of florins, the last figure is read as florins, and all the rest as pounds—thus, 7468 florins is read as £746, 8 florins.

The following examples, in the various rules of Arithmetic, will shew the method of calculating in decimal money:—

Examples.

1. Add together 43 pounds, 7 florins, 83 cents; 126 pounds, 2 florins, 74 cents; 342 pounds, 9 florins, 86 cents.

£	Florins.	Cents.		
43	7	83		£43 · 783
126	2	74	or,	126 · 274
342	9	86		342 · 986
513	0	43		£513 · 043 *Answer.*

2. Subtract from 4978 pounds, 9 florins, 87 cents; 3785 pounds, 8 florins, 98 cents.

£	Florins.	Cents.		
4978	9	87		£4978 · 987
3785	8	98	or,	3785 · 898
1193	0	89		£1193 · 089 *Answer.*

3. Multiply 2354 pounds, 6 florins, 49 cents, by 5.

$$\begin{array}{r} £2354 \cdot 649 \\ 5 \\ \hline £11773 \cdot 245 \end{array}$$ *Answer.*

4. Divide 8796 pounds, 8 florins, 96 cents, by 8.

$$\begin{array}{r} 8\,)\,£8796 \cdot 896 \\ \hline £1099 \cdot 612 \end{array}$$ *Answer.*

5. What is the price of 35 yards of cloth, at 7 florins 25 cents per yard (= 14*s.* 6*d.*)?

·725 35 3625 2175 £25·375 *Ans.*	Here the 7 florins 25 cents are written as 725 cents, and being multiplied by 35, amount to 25375 cents. The decimal point being placed before the third figure from the right, the three last figures express cents (or florins and cents), and all the rest pounds. The answer may be read either as £25·375 cents, or as £25, 3 florins, 75 cents.

6. What is the price of 42 quarters of wheat, at £2, 9 florins, 50 cents, per quarter (= £2, 19*s.*)?

£2·950 42 5900 11800 £123·900 *Ans.*	Here the three last figures are cents, and all the rest pounds, the decimal point being placed between them. The answer may be read either as £123, 900 cents; or as £123, 9 florins.

7. What is the interest for 1 year on £95, 8 florins, 45 cents, at 4 per cent.?

£95·845 4 383·380 *Ans.* £3·833$\frac{80}{100}$	To divide ordinary numbers by 100, the two last figures are struck off. In decimals, the division is accomplished merely by removing the decimal point two places to the *left*, and then writing the two last figures as a fraction with the divisor, 100, placed below.

Exercises.

1. Add £2, 5 florins, 33 cents; £4, 9 florins, 86 cents; £7, 3 cents; £8, 6 florins, 75 cents, ... Ans. 23·197
2. Subtract £136, 9 florins, 29 cents, from £254, 3 florins, 18 cents, Ans. 117·889
3. Multiply £25, 9 florins, 27 cents, by 3, 5, 6, 8, Ans. £77·781; £129·635; £155·562; £207·416
4. Divide £75, 2 florins, 36 cents, by 2, 4, 7, 8, Ans. £37·618; £18·809; £10·748; £9·404½

What is the price of the following:—

5.	37 yards of cloth, at £1·075 per yard, . . .	Ans.	£ 39·775
6.	132 " " £·904 " . . .	"	119·328
7.	435 lbs. of sugar, at 17 cents per lb. . . .	"	7·395
8.	122½ yards of satin, at £1·326 per yard, .	"	162·435
9.	620 " of cotton, at 76 cents, " . . .	"	47·120
10.	336 lbs. of tea, at 2 florins, 35 cents per lb. .	"	78·960
11.	538 " of sugar, at 13 cents " .	"	6·994
12.	18 quarters of wheat, at £2·436 per quarter,	"	43·848

What is the interest for 1 year on:—

13.	£378,	4 florins,	37 cents,	at 2 per cent. . .	Ans.	£ 7·568 $\frac{74}{100}$
14.	296,	5 "	24 "	2½ " . . .	"	7·413 $\frac{10}{100}$
15.	425,	6 "	42 "	3 " . . .	"	12·769 $\frac{26}{100}$
16.	369,	4 "	82 "	4 " . . .	"	14·779 $\frac{28}{100}$
17.	654,	9 "	76 "	5 " . . .	"	32·748 $\frac{80}{100}$

SHILLINGS, PENCE, AND FARTHINGS may be converted into decimal money; and DECIMAL MONEY into shillings, &c. by the following rules: they give the answer within a farthing or a cent of the exact value.

I. To CONVERT SHILLINGS, PENCE, AND FARTHINGS INTO DECIMAL MONEY.

RULE.—Reckon *half* the number of shillings as so many florins; and if the shillings are an odd number, reckon the 1*s.* over as 50 cents; convert the pence into farthings, which, with any farthings in the given sum, reckon as so many cents—adding one more for every 24; then add the whole for the answer.

Example.—Convert 7*s.* 8¼*d.* into decimal money.

	Florins.	Cents.
7*s.* . . . . =	3	50
8¼*d.* = 33 farthings, and adding 1 =		34
	3	84 *Ans.*

Here 1 is added to the 33 farthings, there being *once* 24 in 33.

Exercises.—Convert into decimal money:—

					Florins.	Cents.						Florins.	Cents.
1.	£0	6	5	. Ans.	3	20	5.	£0	17	6½	. Ans.	8	77
2.	0	7	8½	. "	3	85	6.	0	12	9¼	. "	6	38
3.	0	13	3	. "	6	62	7.	0	16	10	. "	8	41
4.	0	11	7¾	. "	5	82	8.	0	19	4	. "	9	66

II. To CONVERT DECIMAL MONEY INTO SHILLINGS, PENCE, AND FARTHINGS.

RULE.—Reckon *twice* the number of florins as so many shillings; and reckon the cents as so many farthings—less 1 for every 25; then add the whole for the answer.

Example.—Convert 5 florins, 42 cents, into shillings, &c.

5 florins, ... =	10*s.*	
42 cents, less 1 = 41 farthings, =		10¼*d.*
	10*s.*	10¼*d.* *Ans.*

Here 1 is deducted from 42 cents, there being once 25 in 42.

Exercises.—Convert into shillings, &c.:—

	Florins.	Cents.						Florins.	Cents.				
1.	0	62	. Ans. £0	1	3		4.	7	92	. Ans. 0	15	10¼	
2.	1	43	. " 0	2	10½		5.	8	87	. " 0	17	9	
3.	5	06	. " 0	10	1½		6.	9	65	. " 0	19	3¾	

THE END.

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